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On the Luni-Solar Variations of Magnetic Declination and Horizontal Force at Bombay, and of Declination at Trevandrum

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PHILOSOPHICAL TRANSACTIONS.

I. *On the Luni-Solar Variations of Magnetic Declination and Horizontal Force at Bombay, and of Declination at Trevandrum.*

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[PLATES 1–5.]

IN the early attempts to investigate the influence of the moon upon terrestrial magnetism, the observations dealt with extended over periods so limited that little was possible beyond determining the average character of the lunar diurnal variation. This was mainly because magnetic disturbance tends—and especially in extra-tropical regions—to mask the minute variations that depend upon the moon. The series of observations made at the Colába Observatory, Bombay, and discussed in the present paper, extending over twenty-five years in the case of the declination and over twenty-six and a half years in the case of the horizontal force, possesses therefore the double advantage of being originally affected by only the relatively small disturbances of a tropical station, and of being lengthy enough to secure an approximate elimination of such disturbance as is involved in it, even by combination of portions only of the whole body of observations.

2. The instruments used at Colába were made by GRUBB, of Dublin, and are like those described in the report (of 1840) of the Committee of Physics of the Royal Society, the magnets being fifteen inches long. An account of them and of their history will be found in the 'Appendices to the Bombay Magnetical and Meteorological Observations, 1879 to 1882,' pages [84] and [138]: and to this account reference may be made for particulars as to the adjustments and determination of scale coefficients of both the declination and horizontal force magnetometers, and as to the determination of the temperature coefficient of the latter instrument. The following extract is, however, given in full, the matter of it being essential to an understanding of the principal object of this paper.

3. *Method of Reduction of the Observations.*—The mode of treatment adopted is that which was introduced by General Sir EDWARD SABINE, and which is described by him in the 'Proceedings of the Royal Society,' vol. 10, pp. 624–626, in the following words:—"The hourly directions of the magnet are entered in monthly

Tables, having the days of the month in successive horizontal lines, and the hours of the day in vertical columns. The 'means' of the entries in each vertical column indicate the mean direction of the magnet at the different hours of the month to which the Table belongs, and have received the name of 'first normals.' On inspecting any such monthly Table, it is at once seen that a considerable portion of the entries in the several columns differ considerably from their respective means or first normals, and must be regarded as 'disturbed observations.' The laws of their relative frequency and amount of disturbance in different years, months, and hours are then sought out by separating for that purpose a sufficient body of the most disturbed observations, computing the amount of departure in each case from the normal of the same month and hour, and arranging the amounts in annual, monthly, and hourly tables. In making these computations, the first normals require to be themselves corrected by the omission in each vertical column of the entries noted as disturbed, and by taking fresh means, representing the normals of each month and hour after this omission, and therefore uninfluenced by the larger disturbances. These new means have received the name of 'final normals,' and may be defined as being the mean directions of the magnet in every month and every hour after the omission from the record of every entry which differed from the mean by a certain amount either in excess or in defect.

"In this process there is nothing indefinite, and nothing arbitrary save the assignment of the particular amount of difference from the normal which shall be held to constitute the measure of a large disturbance, and which, for distinction's sake, we may call 'the separating value.' It must be an amount which will separate a sufficient body of disturbed observations to permit their laws to be satisfactorily ascertained, but in other respects its precise value is of minor significancy; and the limits within which a selection may be made, without materially affecting the results, are usually by no means narrow, for it has been found experimentally on several occasions that the ratios by which the periodical variations of disturbance in different years, months, and hours are characterised and expressed do not undergo any material change by even considerable differences in the amount of the separating value. The separating value must necessarily be larger at some stations than at others, because the absolute magnitude of the disturbance variation itself is very different in different parts of the globe, as well as its comparative magnitude in relation to the more regular solar diurnal variation, but it must be a constant quantity throughout at one and the same station, or it will not truly show the relative proportion of disturbance in different years and different months." The words "directions of the magnet" in the extract must be taken when applied to the Colába declination observations to imply those directions as expressed either by the original scale readings or those readings converted into minutes of easterly declination; and, when applied to the horizontal force observations, to imply the directions as expressed by the original scale readings after reduction to a uniform temperature.

4. The period of the declination observations is from 1846·0 to 1871·0, and that of the horizontal force observations from 1846·5 to 1873·0. With the exception of Sundays and eight or ten complete days in each year, the observations were taken continuously at hourly intervals throughout these periods. The entries in the monthly declination Tables of the years 1846 to 1865 were in minutes of arc, in those of the years 1866 to 1870 in original scale readings, and the "separating value" made

use of was 1′·4 or its equivalent ·204 scale divisions. The separating value adopted for the horizontal force was the equivalent in scale reading of ·000334 of a C.G.S. unit of force.

5. Continuing now the extract—from the monthly Tables already described in it, "all observations having been thrown out which deviated from the 'Final Normal' of each hour by more than the amount of the separating value, new Tables were formed in which each observation was substituted by its excess above the Final Normal of its own hour; or rather, by that excess, *plus* a constant round number. Practically, the Final Normal was diminished by the round number, and the difference being then taken between the number found and each observation in the same hour column, all the differences have the *positive* sign, and thus the inconvenience of dealing with *positive* and *negative* numbers is avoided. On the new Tables the observation at the solar hour of each day which was nearest to the time of the moon's crossing the meridian of Bombay (from East to West) was marked with a figure 0 to indicate that that observation must be placed to the 0th hour of the lunar day in Tables having the lunar hours, from 0 to 23, marked in consecutive order at the top of the several columns. If only twenty-three observations intervened between two marked ones, they were entered in the Table consecutively as forming a complete lunar day; but, as twenty-four observations generally intervened, the two which were in the nearest correspondence to the same lunar hour were combined together, and the mean of the two treated as a single observation." The times of New Moon, First Quarter, Full Moon, and Last Quarter—taken from the Nautical Almanac, and duly corrected for difference of longitude between Greenwich and Bombay, or Göttingen*—were also marked on the margin of the sheet opposite to the several corresponding solar days; for the groups of days, two before and two after these times respectively, the moon was regarded as at the several quarters; and during the intervening days the designations of the moon's phases were one-eighth, three-eighths, five-eighths, and seven-eighths respectively. The entries in the Tables of lunar differences, thus marked, were now distributed, in full lunar days, amongst thirty-two new Tables, called lunar abstracts, of which there was one for each eighth of a cycle of the moon's phases in each quarter of the year. The whole of a lunar day's differences were entered in the lunar abstracts under that variety of phase to which the greater half of the day belonged. Whenever the number of undisturbed observations on a lunar day was less than twelve the whole day's differences were rejected. When all the differences of each category had been entered on their respective lunar Tables, the hourly means were taken on these Tables, and the excess was then found of each of these means above the mean of all the hours; finally, these excesses were converted into force equivalents. The series of hourly excesses thus found may, in a sufficiently extended inquiry, be taken to represent the lunar diurnal variation of declination or horizontal force when the sun and moon have the positions indicated by that particular Table.

6. The following Tables show, for each magnetic element, the converted hourly excesses found on each of the thirty-two lunar abstracts:—†

* Until 1866·0 the records were made at hours of Göttingen Astronomical time; since that date, at hours of Bombay Civil time. All the results tabulated or graphically represented in this paper have reference, however, to the lunar day or the solar astronomical day at the place of observation.

† The calculations having been made also for the summer and winter half-years, and for the full year, their results for these periods are also shown in the Tables.

TABLE I.—Declination at Bombay, showing the Mean Lunar Diurnal Variation at New Moon and One-eighth Phase in each Quarter of the Year.

Phase of moon.	New moon.							One-eighth.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	+·000016	·000000	−·000012	−·000021	−·000018	+·000010	−·000004	+·000006	−·000002	−·000030	−·000007	−·000019	+·000002	−·000008
1	+·000018	−·000006	−·000018	−·000017	−·000023	+·000012	−·000006	+·000014	−·000002	−·000012	−·000007	−·000007	+·000003	−·000002
2	+·000008	−·000006	−·000008	−·000009	−·000010	+·000002	−·000004	+·000014	+·000004	−·000004	+·000015	+·000001	+·000013	+·000008
3	+·000006	−·000004	−·000002	+·000013	+·000003	+·000003	+·000003	+·000005	+·000006	+·000006	+·000010	+·000008	+·000005	+·000006
4	−·000004	−·000004	−·000002	+·000021	+·000008	−·000004	+·000003	−·000004	+·000005	−·000001	+·000007	+·000003	+·000001	+·000002
5	−·000008	−·000004	+·000017	+·000015	+·000015	−·000006	+·000005	−·000003	−·000002	+·000004	+·000002	+·000002	−·000002	·000000
6	−·000012	−·000007	+·000014	+·000008	+·000010	−·000009	+·000001	−·000004	−·000002	−·000002	·000000	−·000001	−·000004	−·000002
7	−·000011	−·000011	+·000003	+·000004	+·000001	−·000009	−·000003	−·000005	−·000001	·000000	·000000	·000000	−·000004	−·000001
8	−·000006	−·000011	+·000002	+·000006	+·000002	−·000007	−·000002	−·000002	−·000004	·000000	+·000001	·000000	−·000003	−·000001
9	−·000003	−·000005	+·000002	+·000001	+·000002	−·000004	−·000001	−·000003	+·000001	·000000	−·000003	−·000001	−·000002	−·000001
10	−·000001	+·000001	+·000003	+·000002	+·000002	·000000	+·000001	·000000	+·000005	·000000	−·000002	−·000002	+·000003	+·000001
11	+·000004	+·000003	+·000002	+·000001	+·000001	+·000003	+·000003	+·000003	+·000008	−·000001	−·000004	−·000003	+·000005	+·000002
12	+·000004	+·000015	+·000003	−·000002	+·000002	+·000008	+·000005	+·000004	+·000006	+·000001	·000000	−·000001	+·000006	+·000003
13	+·000006	+·000004	−·000001	−·000001	−·000001	+·000005	+·000002	+·000006	+·000001	−·000003	+·000003	−·000003	+·000006	+·000002
14	+·000004	+·000010	·000000	+·000003	+·000001	+·000007	+·000004	+·000006	·000000	−·000002	+·000003	−·000003	+·000006	+·000002
15	+·000005	+·000012	+·000002	·000000	+·000001	+·000009	+·000005	+·000005	−·000001	+·000006	+·000004	+·000003	+·000004	+·000004
16	+·000001	+·000001	+·000005	+·000004	+·000003	+·000002	+·000003	+·000001	−·000004	+·000014	+·000001	+·000010	−·000004	+·000003
17	−·000004	+·000004	+·000010	+·000005	+·000008	−·000001	+·000004	−·000008	−·000002	+·000014	+·000013	+·000015	−·000006	+·000005
18	−·000016	+·000003	+·000012	+·000003	+·000010	−·000010	+·000001	−·000008	−·000009	+·000021	+·000006	+·000014	−·000008	+·000003
19	−·000009	+·000007	+·000012	+·000008	+·000014	−·000005	+·000005	−·000010	+·000005	+·000010	−·000004	+·000006	−·000005	+·000001
20	−·000001	−·000003	−·000003	+·000006	+·000004	−·000006	·000000	−·000006	−·000003	+·000010	−·000003	+·000006	−·000007	·000000
21	−·000008	+·000003	−·000012	−·000012	−·000011	−·000005	−·000007	−·000004	·000000	−·000002	−·000003	−·000002	−·000003	−·000002
22	−·000006	+·000005	−·000009	−·000022	−·000013	−·000002	−·000007	+·000001	−·000005	−·000022	−·000010	−·000017	−·000001	−·000009
23	+·000008	−·000006	−·000018	−·000018	−·000021	+·000003	−·000008	+·000001	−·000013	−·000017	−·000013	−·000021	−·000001	−·000010
Number of lunar days' observations in abstract	239	250	264	249	520	482	1002	198	198	200	210	408	398	806

TABLE II.—Declination at Bombay, showing the Mean Lunar Diurnal Variation at First Quarter and Three-eighths Phase in each Quarter of the Year.

Phase of moon.	First quarter.							Three-eighths.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	+ ·000018	+ ·000003	− ·000010	− ·000002	− ·000007	+ ·000011	+ ·000002	+ ·000008	+ ·000001	− ·000001	− ·000001	− ·000001	+ ·000004	+ ·000002
1	+ ·000012	+ ·000003	− ·000003	·000000	− ·000002	+ ·000007	+ ·000002	+ ·000005	·000000	·000000	− ·000001	− ·000001	+ ·000003	+ ·000001
2	+ ·000006	+ ·000004	+ ·000001	+ ·000001	·000000	+ ·000004	+ ·000002	− ·000001	− ·000002	− ·000005	+ ·000007	− ·000001	+ ·000001	·000000
3	− ·000002	− ·000002	+ ·000003	+ ·000002	+ ·000002	− ·000002	·000000	− ·000002	− ·000003	− ·000004	+ ·000004	− ·000002	·000000	− ·000001
4	+ ·000002	− ·000002	·000000	+ ·000006	− ·000001	+ ·000003	+ ·000001	− ·000002	− ·000007	− ·000007	+ ·000002	− ·000004	− ·000003	− ·000003
5	− ·000001	− ·000004	+ ·000001	− ·000004	− ·000001	− ·000004	− ·000002	− ·000010	− ·000012	− ·000008	− ·000005	− ·000006	− ·000010	− ·000008
6	− ·000006	+ ·000001	·000000	− ·000003	− ·000002	− ·000003	− ·000002	− ·000011	− ·000007	− ·000005	− ·000002	− ·000003	− ·000009	− ·000006
7	− ·000002	+ ·000001	− ·000003	− ·000008	− ·000004	− ·000003	− ·000003	− ·000011	− ·000005	− ·000003	+ ·000002	− ·000001	− ·000007	− ·000004
8	− ·000006	·000000	− ·000005	− ·000008	− ·000005	− ·000005	− ·000005	− ·000010	+ ·000003	·000000	+ ·000003	+ ·000003	− ·000004	− ·000001
9	− ·000009	− ·000002	− ·000006	− ·000007	− ·000004	− ·000008	− ·000006	− ·000007	+ ·000004	− ·000004	− ·000004	− ·000002	− ·000004	− ·000003
10	− ·000005	+ ·000001	− ·000005	− ·000006	− ·000005	− ·000003	− ·000004	− ·000002	+ ·000011	− ·000004	− ·000012	− ·000002	− ·000001	− ·000002
11	+ ·000004	+ ·000011	− ·000008	− ·000011	− ·000008	+ ·000005	− ·000002	+ ·000001	+ ·000001	− ·000010	− ·000013	− ·000009	− ·000001	− ·000005
12	+ ·000006	+ ·000011	− ·000012	− ·000009	− ·000009	+ ·000006	− ·000001	+ ·000011	+ ·000001	− ·000014	− ·000007	− ·000011	+ ·000006	− ·000003
13	+ ·000006	+ ·000001	− ·000010	− ·000002	− ·000009	+ ·000006	− ·000002	+ ·000022	+ ·000007	− ·000013	− ·000007	− ·000009	+ ·000014	+ ·000002
14	+ ·000006	− ·000001	− ·000006	+ ·000006	·000000	+ ·000001	+ ·000001	+ ·000011	+ ·000008	− ·000009	+ ·000005	− ·000004	+ ·000011	+ ·000004
15	+ ·000008	− ·000004	+ ·000004	+ ·000014	+ ·000007	+ ·000003	+ ·000005	+ ·000020	− ·000009	+ ·000001	+ ·000001	·000000	+ ·000007	+ ·000003
16	− ·000002	− ·000004	+ ·000020	+ ·000023	+ ·000015	+ ·000002	+ ·000008	+ ·000006	− ·000003	+ ·000010	+ ·000011	+ ·000009	+ ·000004	+ ·000006
17	− ·000004	− ·000016	+ ·000013	+ ·000019	+ ·000014	− ·000009	+ ·000002	− ·000006	+ ·000011	+ ·000024	+ ·000016	+ ·000020	+ ·000003	+ ·000011
18	− ·000015	− ·000009	+ ·000013	+ ·000013	+ ·000011	− ·000011	·000000	− ·000002	+ ·000008	+ ·000030	+ ·000008	+ ·000019	+ ·000004	+ ·000012
19	− ·000015	− ·000004	+ ·000011	+ ·000001	+ ·000008	− ·000012	− ·000002	− ·000019	+ ·000002	+ ·000019	+ ·000007	+ ·000013	− ·000007	+ ·000003
20	− ·000026	− ·000005	+ ·000013	+ ·000001	+ ·000009	− ·000017	− ·000004	− ·000011	+ ·000001	+ ·000006	− ·000005	+ ·000002	− ·000006	− ·000002
21	·000000	+ ·000004	+ ·000002	− ·000006	− ·000001	·000000	− ·000001	− ·000004	− ·000001	− ·000002	− ·000001	− ·000001	− ·000002	− ·000002
22	+ ·000013	− ·000001	− ·000010	− ·000009	− ·000011	+ ·000006	− ·000003	+ ·000001	− ·000007	− ·000004	− ·000002	− ·000004	− ·000001	− ·000002
23	+ ·000016	+ ·000004	− ·000009	− ·000001	− ·000008	+ ·000012	+ ·000002	+ ·000003	− ·000002	− ·000002	·000000	·000000	·000000	·000000
Number of lunar days' observations in abstract	254	262	264	260	522	518	1040	199	198	213	212	417	405	822

TABLE III.—Declination at Bombay, showing the Mean Lunar Diurnal Variation at Full Moon and Five-eighths Phase in each Quarter of the Year.

Phase of moon.	Full moon.							Five-eighths.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	+ ·000008	·000000	− ·000004	+ ·000002	·000000	+ ·000004	+ ·000002	+ ·000001	+ ·000004	− ·000003	·000000	− ·000001	+ ·000002	+ ·000001
1	+ ·000004	− ·000002	+ ·000002	·000000	+ ·000001	+ ·000001	+ ·000001	+ ·000008	+ ·000009	·000000	+ ·000003	+ ·000003	+ ·000007	+ ·000005
2	+ ·000001	− ·000003	+ ·000003	+ ·000005	+ ·000002	·000000	+ ·000001	+ ·000008	+ ·000002	− ·000004	+ ·000006	+ ·000001	+ ·000004	+ ·000003
3	− ·000006	− ·000007	− ·000001	+ ·000004	·000000	− ·000005	− ·000002	+ ·000010	− ·000007	− ·000001	+ ·000007	+ ·000005	·000000	+ ·000002
4	− ·000007	− ·000006	+ ·000001	+ ·000001	+ ·000001	− ·000006	− ·000003	+ ·000001	− ·000006	+ ·000013	+ ·000005	+ ·000010	− ·000003	+ ·000004
5	− ·000009	− ·000007	+ ·000008	+ ·000005	+ ·000005	− ·000006	·000000	− ·000009	− ·000005	+ ·000015	+ ·000019	+ ·000016	− ·000006	+ ·000006
6	− ·000015	− ·000001	+ ·000006	+ ·000007	+ ·000009	− ·000010	− ·000001	− ·000024	− ·000006	− ·000001	+ ·000009	+ ·000002	− ·000012	− ·000005
7	− ·000011	+ ·000009	+ ·000012	+ ·000009	+ ·000014	− ·000006	+ ·000004	− ·000017	+ ·000004	+ ·000005	+ ·000004	+ ·000004	− ·000007	− ·000001
8	− ·000018	+ ·000004	+ ·000010	+ ·000005	+ ·000007	− ·000007	·000000	− ·000013	·000000	− ·000001	+ ·000001	− ·000001	− ·000006	− ·000003
9	− ·000017	+ ·000008	+ ·000003	− ·000002	+ ·000001	− ·000005	− ·000003	− ·000015	+ ·000007	− ·000005	− ·000002	− ·000005	− ·000003	− ·000004
10	+ ·000001	+ ·000009	− ·000018	− ·000010	− ·000011	+ ·000001	− ·000005	− ·000010	·000000	− ·000013	− ·000017	− ·000016	− ·000004	− ·000010
11	+ ·000009	+ ·000004	− ·000015	− ·000020	− ·000016	+ ·000005	− ·000005	+ ·000008	− ·000002	− ·000020	− ·000006	− ·000015	+ ·000004	− ·000005
12	+ ·000008	+ ·000001	− ·000020	− ·000015	− ·000015	+ ·000002	− ·000006	+ ·000013	− ·000006	− ·000021	− ·000012	− ·000018	+ ·000005	− ·000006
13	+ ·000012	− ·000005	− ·000015	− ·000014	− ·000014	+ ·000004	− ·000005	+ ·000018	− ·000005	− ·000007	− ·000011	− ·000010	+ ·000007	− ·000001
14	+ ·000020	− ·000003	− ·000002	− ·000003	·000000	+ ·000007	+ ·000003	+ ·000015	+ ·000003	+ ·000001	+ ·000001	+ ·000003	+ ·000007	+ ·000005
15	+ ·000010	+ ·000002	+ ·000009	+ ·000011	+ ·000008	+ ·000008	+ ·000008	+ ·000003	+ ·000001	+ ·000006	− ·000001	+ ·000004	+ ·000001	+ ·000002
16	− ·000005	+ ·000003	+ ·000009	+ ·000017	+ ·000011	+ ·000001	+ ·000006	·000000	+ ·000006	+ ·000006	+ ·000004	+ ·000006	+ ·000001	+ ·000004
17	− ·000007	·000000	+ ·000009	+ ·000016	+ ·000010	− ·000001	+ ·000005	+ ·000006	·000000	+ ·000006	− ·000003	+ ·000003	+ ·000001	+ ·000003
18	+ ·000001	− ·000005	+ ·000001	+ ·000007	+ ·000001	+ ·000001	+ ·000001	+ ·000002	− ·000001	+ ·000005	·000000	+ ·000004	·000000	+ ·000002
19	+ ·000006	− ·000002	− ·000003	− ·000007	− ·000006	+ ·000003	− ·000001	− ·000001	− ·000010	+ ·000001	+ ·000003	·000000	− ·000004	− ·000002
20	+ ·000006	− ·000002	·000000	− ·000006	− ·000002	+ ·000001	·000000	− ·000004	− ·000002	·000000	+ ·000003	·000000	− ·000001	− ·000001
21	+ ·000003	− ·000001	+ ·000004	− ·000002	+ ·000001	+ ·000002	+ ·000001	+ ·000002	+ ·000003	+ ·000007	− ·000002	+ ·000004	+ ·000001	+ ·000003
22	+ ·000002	− ·000004	+ ·000001	− ·000003	·000000	− ·000001	− ·000001	− ·000001	+ ·000002	+ ·000005	− ·000001	+ ·000003	·000000	+ ·000001
23	+ ·000004	− ·000001	+ ·000004	+ ·000001	+ ·000004	+ ·000001	+ ·000002	+ ·000003	+ ·000006	+ ·000002	+ ·000001	+ ·000002	+ ·000004	+ ·000003
Number of lunar days' observations in abstract	256	228	254	241	493	486	979	188	192	207	207	406	388	794

TABLE IV.—Declination at Bombay, showing the Mean Lunar Diurnal Variation at Last Quarter and Seven-eighths Phase in each Quarter of the Year.

Phase of moon.	Last quarter.							Seven-eighths.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	+ ·000010	+ ·000003	− ·000007	− ·000010	− ·000008	+ ·000006	− ·000001	+ ·000015	+ ·000008	− ·000016	− ·000015	− ·000013	+ ·000010	− ·000002
1	+ ·000009	− ·000002	− ·000015	− ·000009	− ·000014	+ ·000005	− ·000004	+ ·000017	+ ·000001	− ·000016	− ·000004	− ·000008	+ ·000008	− ·000001
2	+ ·000011	− ·000006	− ·000006	− ·000007	− ·000011	+ ·000007	− ·000002	+ ·000015	− ·000001	− ·000003	+ ·000001	− ·000001	+ ·000008	+ ·000003
3	+ ·000014	− ·000008	+ ·000010	− ·000001	+ ·000004	+ ·000003	+ ·000004	+ ·000007	− ·000005	+ ·000007	+ ·000009	+ ·000005	+ ·000005	+ ·000005
4	·000000	− ·000016	+ ·000010	+ ·000004	+ ·000004	− ·000005	− ·000001	+ ·000006	− ·000005	+ ·000003	+ ·000023	+ ·000007	+ ·000007	+ ·000007
5	− ·000011	+ ·000009	·000000	+ ·000019	+ ·000008	·000000	+ ·000004	− ·000007	+ ·000005	+ ·000001	+ ·000016	+ ·000009	− ·000001	+ ·000004
6	− ·000018	− ·000003	+ ·000010	+ ·000015	+ ·000012	− ·000011	+ ·000001	− ·000017	+ ·000001	+ ·000010	+ ·000019	+ ·000012	− ·000005	+ ·000003
7	− ·000025	− ·000002	+ ·000010	+ ·000032	+ ·000015	− ·000008	+ ·000003	− ·000017	− ·000003	+ ·000007	+ ·000003	+ ·000007	− ·000012	− ·000003
8	− ·000029	− ·000008	+ ·000010	+ ·000005	+ ·000003	− ·000014	− ·000005	− ·000013	− ·000006	+ ·000001	− ·000010	− ·000003	− ·000010	− ·000007
9	− ·000014	+ ·000002	·000000	− ·000005	− ·000002	− ·000007	− ·000005	− ·000004	− ·000006	− ·000006	− ·000004	− ·000005	− ·000005	− ·000005
10	·000000	− ·000001	− ·000017	− ·000009	− ·000013	− ·000001	− ·000007	− ·000001	+ ·000003	− ·000008	− ·000005	− ·000005	·000000	− ·000003
11	+ ·000010	+ ·000001	− ·000011	− ·000009	− ·000008	+ ·000003	− ·000003	− ·000001	+ ·000004	− ·000002	− ·000002	− ·000002	+ ·000001	− ·000001
12	+ ·000011	+ ·000004	− ·000004	+ ·000001	− ·000002	+ ·000008	+ ·000003	·000000	+ ·000007	− ·000001	+ ·000004	+ ·000002	+ ·000004	+ ·000002
13	+ ·000007	+ ·000003	+ ·000001	− ·000001	·000000	+ ·000004	+ ·000002	+ ·000002	·000000	+ ·000002	− ·000002	+ ·000001	+ ·000001	·000000
14	+ ·000005	+ ·000004	+ ·000001	+ ·000005	+ ·000003	+ ·000004	+ ·000003	+ ·000002	+ ·000008	+ ·000007	·000000	+ ·000004	+ ·000005	+ ·000004
15	+ ·000001	− ·000001	− ·000001	+ ·000003	− ·000002	+ ·000002	·000000	·000000	+ ·000002	+ ·000003	+ ·000006	+ ·000005	+ ·000002	+ ·000003
16	+ ·000003	+ ·000006	+ ·000001	·000000	+ ·000001	+ ·000003	+ ·000002	− ·000004	+ ·000008	·000000	+ ·000002	+ ·000003	·000000	+ ·000001
17	·000000	+ ·000006	+ ·000001	− ·000002	·000000	+ ·000002	+ ·000001	− ·000002	+ ·000001	+ ·000001	− ·000003	+ ·000001	− ·000002	− ·000001
18	− ·000003	+ ·000001	+ ·000002	− ·000001	+ ·000002	− ·000003	− ·000001	+ ·000002	− ·000005	+ ·000003	− ·000006	+ ·000001	− ·000003	− ·000002
19	·000000	− ·000004	+ ·000002	− ·000007	·000000	− ·000005	− ·000002	+ ·000002	− ·000003	+ ·000004	− ·000005	+ ·000001	− ·000001	− ·000001
20	+ ·000001	− ·000001	+ ·000003	− ·000001	+ ·000003	− ·000003	·000000	− ·000003	− ·000001	+ ·000012	− ·000004	+ ·000006	− ·000004	+ ·000001
21	+ ·000006	+ ·000002	+ ·000006	− ·000006	+ ·000003	− ·000001	+ ·000001	− ·000004	− ·000003	+ ·000005	·000000	+ ·000001	·000000	·000000
22	+ ·000010	+ ·000001	+ ·000005	− ·000006	+ ·000001	+ ·000004	+ ·000002	·000000	− ·000001	− ·000011	− ·000012	− ·000011	+ ·000001	− ·000006
23	+ ·000011	+ ·000002	− ·000003	− ·000009	− ·000005	+ ·000005	·000000	+ ·000005	− ·000009	− ·000005	− ·000022	− ·000014	·000000	− ·000008
Number of lunar days' observations in abstract	247	255	258	256	515	501	1016	204	198	199	200	402	399	801

TABLE V.—Horizontal Force at Bombay, showing the Mean Lunar Diurnal Variation at New Moon and One-eighth Phase in each Quarter of the Year.

Phase of moon.	New moon.							One-eighth.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	− ·000008	·000000	− ·000007	− ·000009	− ·000008	− ·000003	− ·000006	+ ·000004	− ·000003	+ ·000013	− ·000011	+ ·000005	− ·000005	·000000
1	− ·000016	− ·000005	− ·000009	− ·000001	− ·000010	− ·000006	− ·000008	− ·000009	− ·000007	+ ·000002	− ·000015	− ·000004	− ·000012	− ·000008
2	− ·000028	− ·000009	− ·000009	− ·000003	− ·000007	− ·000017	− ·000012	− ·000004	·000000	+ ·000001	− ·000025	− ·000007	− ·000008	− ·000008
3	− ·000031	− ·000017	− ·000010	− ·000011	− ·000013	− ·000020	− ·000017	− ·000015	− ·000004	− ·000009	− ·000019	− ·000012	− ·000012	− ·000012
4	− ·000019	− ·000021	− ·000008	− ·000013	− ·000012	− ·000017	− ·000015	+ ·000004	− ·000010	− ·000007	− ·000008	− ·000009	− ·000002	− ·000006
5	− ·000010	− ·000009	+ ·000001	·000000	− ·000001	− ·000007	− ·000004	·000000	− ·000008	− ·000007	− ·000009	− ·000012	− ·000001	− ·000006
6	− ·000007	− ·000007	+ ·000005	− ·000005	+ ·000001	− ·000007	− ·000003	− ·000007	− ·000011	− ·000003	− ·000014	− ·000012	− ·000006	− ·000009
7	·000000	− ·000002	+ ·000006	− ·000009	− ·000002	·000000	− ·000001	− ·000009	− ·000016	+ ·000002	− ·000006	− ·000005	− ·000010	− ·000008
8	·000000	·000000	·000000	− ·000006	·000000	− ·000003	− ·000002	− ·000001	− ·000002	− ·000001	+ ·000001	− ·000002	·000000	− ·000001
9	+ ·000006	+ ·000003	− ·000002	+ ·000003	+ ·000001	+ ·000005	+ ·000002	+ ·000002	+ ·000004	− ·000002	+ ·000008	+ ·000004	+ ·000002	+ ·000003
10	+ ·000005	+ ·000006	− ·000010	·000000	− ·000005	+ ·000006	− ·000001	+ ·000006	− ·000002	·000000	+ ·000005	·000000	+ ·000004	+ ·000002
11	+ ·000001	+ ·000001	·000000	− ·000003	+ ·000001	− ·000001	·000000	+ ·000003	− ·000005	+ ·000006	+ ·000002	+ ·000002	+ ·000001	+ ·000001
12	− ·000005	+ ·000006	+ ·000002	+ ·000003	+ ·000005	− ·000002	+ ·000001	− ·000001	− ·000001	− ·000007	+ ·000007	− ·000004	+ ·000002	− ·000001
13	− ·000005	+ ·000003	+ ·000003	+ ·000001	+ ·000004	− ·000003	·000000	− ·000008	− ·000002	− ·000003	+ ·000001	− ·000003	− ·000004	− ·000003
14	− ·000006	− ·000001	+ ·000005	+ ·000003	+ ·000004	− ·000004	·000000	− ·000013	·000000	− ·000016	+ ·000002	− ·000008	− ·000007	− ·000007
15	− ·000002	− ·000002	+ ·000006	+ ·000002	+ ·000005	− ·000002	+ ·000001	− ·000010	− ·000007	− ·000012	+ ·000006	− ·000005	− ·000007	− ·000006
16	− ·000001	− ·000007	+ ·000008	− ·000001	+ ·000006	− ·000007	·000000	− ·000007	− ·000007	− ·000004	+ ·000003	− ·000004	− ·000004	− ·000004
17	+ ·000003	− ·000003	+ ·000008	+ ·000010	+ ·000009	·000000	+ ·000005	− ·000015	− ·000003	− ·000003	+ ·000004	·000000	− ·000009	− ·000005
18	+ ·000012	+ ·000006	+ ·000010	+ ·000013	+ ·000013	+ ·000008	+ ·000010	+ ·000007	− ·000002	− ·000002	+ ·000010	+ ·000001	+ ·000006	+ ·000003
19	+ ·000016	+ ·000007	+ ·000010	+ ·000004	+ ·000013	+ ·000006	+ ·000009	+ ·000012	+ ·000008	+ ·000001	+ ·000008	+ ·000005	+ ·000009	+ ·000007
20	+ ·000023	+ ·000013	·000000	+ ·000001	+ ·000006	+ ·000013	+ ·000009	+ ·000024	+ ·000027	+ ·000010	+ ·000003	+ ·000013	+ ·000019	+ ·000016
21	+ ·000031	+ ·000017	·000000	+ ·000016	+ ·000007	+ ·000025	+ ·000015	+ ·000019	+ ·000029	+ ·000027	+ ·000013	+ ·000021	+ ·000022	+ ·000022
22	+ ·000024	+ ·000010	− ·000009	·000000	− ·000002	+ ·000016	+ ·000006	+ ·000019	+ ·000019	+ ·000010	+ ·000006	+ ·000013	+ ·000014	+ ·000013
23	+ ·000019	+ ·000012	− ·000002	− ·000004	− ·000004	+ ·000017	+ ·000006	+ ·000008	+ ·000011	+ ·000012	+ ·000018	+ ·000015	+ ·000009	+ ·000012
Number of lunar days' observations in abstract	245	240	274	251	527	483	1010	202	196	198	218	402	412	814

TABLE VI.—Horizontal Force at Bombay, showing the Mean Lunar Diurnal Variation at First Quarter and Three-eighths Phase in each Quarter of the Year.

Phase of moon.	First quarter.							Three-eighths.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	+ ·000005	·000000	− ·000002	− ·000002	+ ·000001	+ ·000001	·000000	+ ·000004	+ ·000009	·000000	·000000	+ ·000002	+ ·000004	+ ·000003
1	+ ·000005	+ ·000006	·000000	+ ·000006	+ ·000001	+ ·000009	+ ·000004	+ ·000011	− ·000002	+ ·000002	+ ·000005	+ ·000007	+ ·000002	+ ·000004
2	+ ·000007	+ ·000001	− ·000010	− ·000004	− ·000008	+ ·000005	− ·000002	− ·000001	− ·000005	+ ·000010	− ·000003	+ ·000008	− ·000008	·000000
3	− ·000002	+ ·000001	− ·000009	− ·000007	− ·000009	+ ·000002	− ·000004	+ ·000013	− ·000004	− ·000002	− ·000002	+ ·000003	− ·000001	+ ·000001
4	+ ·000012	− ·000007	− ·000004	− ·000006	− ·000007	+ ·000005	− ·000002	+ ·000003	+ ·000001	− ·000001	− ·000005	− ·000002	+ ·000001	− ·000001
5	+ ·000009	− ·000010	− ·000004	− ·000012	− ·000007	·000000	− ·000004	+ ·000005	+ ·000008	− ·000001	+ ·000007	+ ·000004	+ ·000006	+ ·000005
6	+ ·000002	− ·000002	+ ·000004	− ·000016	− ·000003	− ·000002	− ·000003	+ ·000011	+ ·000012	+ ·000004	+ ·000009	+ ·000007	+ ·000011	+ ·000009
7	+ ·000005	·000000	− ·000002	− ·000011	− ·000005	+ ·000001	− ·000003	+ ·000015	+ ·000012	− ·000002	+ ·000008	+ ·000003	+ ·000013	+ ·000008
8	+ ·000004	− ·000004	− ·000005	− ·000002	− ·000004	+ ·000002	− ·000002	+ ·000019	+ ·000017	− ·000005	+ ·000010	+ ·000003	+ ·000017	+ ·000010
9	+ ·000010	− ·000002	− ·000001	+ ·000001	·000000	+ ·000006	+ ·000002	+ ·000014	+ ·000010	·000000	+ ·000013	+ ·000008	+ ·000010	+ ·000009
10	+ ·000010	+ ·000004	·000000	− ·000002	+ ·000001	+ ·000006	+ ·000003	+ ·000016	+ ·000013	+ ·000007	+ ·000014	+ ·000011	+ ·000014	+ ·000012
11	+ ·000011	+ ·000004	− ·000002	− ·000007	− ·000003	+ ·000007	+ ·000001	+ ·000013	+ ·000010	+ ·000007	+ ·000008	+ ·000009	+ ·000010	+ ·000010
12	+ ·000006	+ ·000001	+ ·000004	− ·000004	·000000	+ ·000004	+ ·000001	− ·000009	− ·000001	− ·000001	+ ·000003	− ·000003	− ·000001	− ·000002
13	− ·000004	− ·000008	− ·000003	− ·000003	− ·000006	− ·000002	− ·000005	− ·000024	− ·000009	+ ·000003	− ·000010	− ·000008	− ·000011	− ·000010
14	− ·000015	− ·000009	− ·000010	− ·000008	− ·000010	− ·000009	− ·000011	− ·000025	− ·000022	− ·000002	− ·000019	− ·000014	− ·000020	− ·000017
15	− ·000029	− ·000022	− ·000013	− ·000006	− ·000011	− ·000023	− ·000018	− ·000029	− ·000025	+ ·000001	− ·000021	− ·000013	− ·000023	− ·000018
16	− ·000022	− ·000016	− ·000001	+ ·000009	+ ·000005	− ·000020	− ·000008	− ·000025	− ·000014	− ·000002	− ·000015	− ·000006	− ·000021	− ·000014
17	− ·000018	− ·000016	− ·000012	+ ·000011	− ·000002	− ·000015	− ·000009	− ·000027	− ·000001	− ·000018	− ·000004	− ·000011	− ·000015	− ·000013
18	− ·000014	− ·000002	− ·000001	+ ·000014	+ ·000008	− ·000009	− ·000001	− ·000007	− ·000003	− ·000011	− ·000007	− ·000010	− ·000005	− ·000007
19	− ·000010	+ ·000004	+ ·000006	+ ·000022	+ ·000013	− ·000002	+ ·000005	− ·000001	− ·000009	− ·000007	+ ·000004	− ·000004	− ·000002	− ·000003
20	+ ·000010	+ ·000009	+ ·000012	+ ·000002	+ ·000008	+ ·000009	+ ·000008	+ ·000009	− ·000004	− ·000002	+ ·000009	+ ·000004	+ ·000003	+ ·000003
21	+ ·000010	+ ·000027	+ ·000019	+ ·000012	+ ·000018	+ ·000016	+ ·000016	+ ·000006	− ·000004	+ ·000004	− ·000003	·000000	+ ·000002	+ ·000001
22	+ ·000010	+ ·000017	+ ·000020	+ ·000016	+ ·000020	+ ·000011	+ ·000015	+ ·000006	+ ·000004	+ ·000003	+ ·000001	+ ·000001	+ ·000007	+ ·000004
23	+ ·000009	+ ·000015	+ ·000007	+ ·000001	+ ·000009	+ ·000008	+ ·000008	+ ·000014	+ ·000015	+ ·000005	·000000	+ ·000006	+ ·000011	+ ·000008
Number of lunar days' observations in abstract	263	252	272	268	541	514	1055	205	198	220	209	416	416	832

TABLE VII.—Horizontal Force at Bombay, showing the Mean Lunar Diurnal Variation at Full Moon and Five-eighths Phase in each Quarter of the Year.

Phase of moon.	Full moon.							Five-eighths.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	+ ·000002	− ·000006	·000000	− ·000008	+ ·000001	− ·000007	− ·000003	·000000	− ·000018	+ ·000001	− ·000002	·000000	− ·000009	− ·000004
1	+ ·000006	− ·000008	− ·000003	·000000	− ·000001	− ·000001	− ·000001	− ·000003	− ·000009	− ·000003	− ·000004	− ·000002	− ·000006	− ·000004
2	− ·000006	− ·000007	− ·000003	− ·000010	− ·000005	− ·000008	− ·000006	− ·000012	− ·000007	− ·000005	− ·000007	− ·000004	− ·000010	− ·000007
3	− ·000007	− ·000008	− ·000003	− ·000005	− ·000003	− ·000007	− ·000005	− ·000019	− ·000007	+ ·000004	− ·000003	·000000	− ·000012	− ·000006
4	− ·000003	·000000	+ ·000003	− ·000004	+ ·000002	− ·000004	− ·000001	− ·000012	− ·000007	+ ·000005	− ·000005	·000000	− ·000009	− ·000004
5	− ·000001	− ·000003	− ·000002	+ ·000007	+ ·000005	− ·000004	+ ·000001	·000000	·000000	+ ·000015	+ ·000004	+ ·000005	+ ·000006	+ ·000006
6	+ ·000001	+ ·000009	·000000	+ ·000003	+ ·000004	+ ·000002	+ ·000003	+ ·000014	− ·000007	+ ·000002	− ·000002	− ·000001	+ ·000005	+ ·000002
7	+ ·000002	+ ·000010	+ ·000003	− ·000001	+ ·000002	+ ·000005	+ ·000004	+ ·000029	+ ·000020	+ ·000007	+ ·000011	+ ·000006	+ ·000027	+ ·000017
8	+ ·000019	+ ·000011	+ ·000002	+ ·000005	+ ·000003	+ ·000016	+ ·000009	+ ·000036	+ ·000021	+ ·000005	+ ·000012	+ ·000006	+ ·000031	+ ·000019
9	+ ·000025	+ ·000015	+ ·000009	+ ·000018	+ ·000011	+ ·000022	+ ·000017	+ ·000033	+ ·000010	+ ·000001	+ ·000019	+ ·000008	+ ·000024	+ ·000016
10	+ ·000027	+ ·000014	+ ·000001	+ ·000010	+ ·000004	+ ·000022	+ ·000013	+ ·000023	+ ·000020	− ·000002	+ ·000009	+ ·000006	+ ·000019	+ ·000013
11	+ ·000011	+ ·000005	+ ·000003	+ ·000006	+ ·000001	+ ·000011	+ ·000006	+ ·000010	+ ·000022	− ·000006	·000000	− ·000002	+ ·000015	+ ·000007
12	− ·000007	+ ·000009	+ ·000007	− ·000004	+ ·000005	− ·000002	+ ·000001	+ ·000005	+ ·000017	− ·000009	+ ·000011	+ ·000004	+ ·000007	+ ·000006
13	− ·000008	− ·000001	+ ·000004	− ·000002	·000000	− ·000004	− ·000002	+ ·000007	+ ·000005	− ·000011	+ ·000014	− ·000001	+ ·000009	+ ·000004
14	− ·000020	− ·000011	− ·000007	− ·000004	− ·000011	− ·000009	− ·000010	− ·000011	+ ·000002	− ·000009	− ·000005	− ·000009	− ·000003	− ·000006
15	− ·000024	− ·000009	− ·000004	− ·000003	− ·000007	− ·000012	− ·000010	− ·000023	·000000	− ·000007	− ·000007	− ·000007	− ·000012	− ·000009
16	− ·000013	− ·000007	− ·000002	− ·000004	− ·000004	− ·000009	− ·000006	− ·000017	+ ·000003	− ·000007	− ·000004	− ·000005	− ·000009	− ·000006
17	− ·000003	− ·000001	·000000	− ·000003	− ·000002	− ·000001	− ·000001	− ·000015	− ·000005	− ·000002	− ·000006	− ·000002	− ·000012	− ·000007
18	− ·000007	− ·000009	− ·000002	− ·000006	− ·000004	− ·000007	− ·000005	− ·000017	− ·000004	+ ·000006	− ·000003	+ ·000001	− ·000011	− ·000005
19	− ·000001	− ·000009	+ ·000002	·000000	+ ·000004	− ·000007	− ·000001	− ·000016	− ·000018	+ ·000004	− ·000017	− ·000003	− ·000019	− ·000011
20	− ·000001	− ·000010	·000000	− ·000001	·000000	− ·000005	− ·000003	− ·000005	− ·000009	+ ·000006	− ·000009	+ ·000002	− ·000010	− ·000004
21	− ·000002	+ ·000005	− ·000008	− ·000003	− ·000002	− ·000001	− ·000002	− ·000007	− ·000010	+ ·000001	− ·000008	− ·000002	− ·000009	− ·000005
22	+ ·000004	·000000	− ·000005	− ·000001	− ·000005	+ ·000004	·000000	+ ·000004	− ·000008	+ ·000004	− ·000003	− ·000001	− ·000001	·000000
23	+ ·000009	·000000	− ·000003	+ ·000005	− ·000001	+ ·000007	+ ·000003	− ·000001	− ·000006	+ ·000001	− ·000006	− ·000002	− ·000004	− ·000003
Number of lunar days' observations in abstract	253	230	258	241	501	481	982	197	188	209	207	412	389	801

TABLE VIII.—Horizontal Force at Bombay, showing the Mean Lunar Diurnal Variation at Last Quarter and Seven-eighths Phase in each Quarter of the Year.

Phase of moon.	Last quarter.							Seven-eighths.						
Bombay lunar hours.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.	November to January.	February to April.	May to July.	August to October.	April to September.	October to March.	Year.
0	− ·000008	− ·000004	− ·000007	+ ·000001	− ·000005	− ·000004	− ·000004	− ·000013	− ·000012	− ·000012	+ ·000002	− ·000010	− ·000008	− ·000009
1	− ·000006	·000000	− ·000007	− ·000006	− ·000008	− ·000003	− ·000005	− ·000025	− ·000025	− ·000018	+ ·000004	− ·000011	− ·000021	− ·000016
2	− ·000018	− ·000002	− ·000002	− ·000001	− ·000002	− ·000010	− ·000006	− ·000045	− ·000026	− ·000020	− ·000013	− ·000019	− ·000034	− ·000027
3	− ·000023	+ ·000006	+ ·000013	− ·000016	+ ·000005	− ·000016	− ·000005	− ·000043	− ·000021	− ·000016	− ·000017	− ·000020	− ·000030	− ·000025
4	− ·000023	+ ·000003	·000000	− ·000006	+ ·000003	− ·000017	− ·000007	− ·000030	− ·000029	− ·000007	− ·000021	− ·000014	− ·000030	− ·000022
5	− ·000004	− ·000003	+ ·000009	− ·000008	+ ·000005	− ·000008	− ·000001	− ·000013	− ·000028	− ·000017	− ·000012	− ·000015	− ·000020	− ·000017
6	+ ·000014	+ ·000007	− ·000002	− ·000011	− ·000002	+ ·000006	+ ·000002	+ ·000001	− ·000007	− ·000022	− ·000018	− ·000022	− ·000001	− ·000011
7	+ ·000032	+ ·000003	− ·000004	− ·000003	·000000	+ ·000014	+ ·000007	− ·000003	+ ·000003	− ·000011	+ ·000002	− ·000003	− ·000002	− ·000003
8	+ ·000028	− ·000005	+ ·000010	+ ·000002	+ ·000008	+ ·000010	+ ·000009	+ ·000020	+ ·000011	− ·000004	+ ·000001	− ·000002	+ ·000016	+ ·000006
9	+ ·000026	− ·000002	+ ·000008	+ ·000006	+ ·000004	+ ·000015	+ ·000009	+ ·000020	·000000	− ·000002	+ ·000009	+ ·000002	+ ·000011	+ ·000007
10	+ ·000021	− ·000003	+ ·000002	− ·000009	− ·000003	+ ·000009	+ ·000003	+ ·000016	·000000	+ ·000003	+ ·000009	+ ·000002	+ ·000012	+ ·000007
11	+ ·000009	+ ·000002	+ ·000006	− ·000001	+ ·000003	+ ·000005	+ ·000004	+ ·000017	+ ·000002	·000000	+ ·000014	+ ·000004	+ ·000013	+ ·000009
12	+ ·000005	− ·000011	− ·000003	+ ·000003	− ·000003	·000000	− ·000001	+ ·000017	+ ·000022	+ ·000007	+ ·000005	+ ·000009	+ ·000016	+ ·000012
13	− ·000001	− ·000003	− ·000004	+ ·000004	− ·000002	·000000	− ·000001	+ ·000021	+ ·000022	+ ·000008	+ ·000008	+ ·000009	+ ·000020	+ ·000014
14	− ·000003	+ ·000002	+ ·000002	+ ·000002	+ ·000002	·000000	+ ·000001	+ ·000004	+ ·000019	+ ·000012	+ ·000007	+ ·000010	+ ·000010	+ ·000010
15	− ·000009	+ ·000002	− ·000001	+ ·000006	+ ·000001	− ·000003	·000000	·000000	+ ·000019	+ ·000008	− ·000001	+ ·000005	+ ·000007	+ ·000006
16	− ·000007	− ·000002	+ ·000002	+ ·000009	+ ·000002	− ·000001	+ ·000001	+ ·000011	+ ·000002	+ ·000011	+ ·000001	+ ·000006	+ ·000006	+ ·000006
17	− ·000011	− ·000007	− ·000002	+ ·000004	− ·000001	− ·000008	− ·000004	+ ·000009	+ ·000004	+ ·000005	+ ·000012	+ ·000009	+ ·000006	+ ·000007
18	+ ·000003	·000000	− ·000004	+ ·000003	− ·000001	+ ·000001	·000000	·000000	+ ·000014	+ ·000012	+ ·000004	+ ·000010	+ ·000004	+ ·000007
19	− ·000006	+ ·000011	− ·000006	+ ·000007	·000000	+ ·000002	+ ·000001	+ ·000001	+ ·000012	+ ·000012	+ ·000002	+ ·000010	+ ·000002	+ ·000006
20	− ·000003	+ ·000010	− ·000003	− ·000004	− ·000004	+ ·000004	·000000	+ ·000006	+ ·000008	+ ·000019	+ ·000005	+ ·000012	+ ·000007	+ ·000009
21	− ·000001	+ ·000007	− ·000004	+ ·000007	− ·000001	+ ·000005	+ ·000002	+ ·000010	+ ·000010	+ ·000017	+ ·000006	+ ·000012	+ ·000010	+ ·000011
22	− ·000006	+ ·000002	− ·000004	+ ·000008	− ·000001	+ ·000001	·000000	+ ·000005	+ ·000007	+ ·000015	+ ·000005	+ ·000010	+ ·000005	+ ·000008
23	− ·000015	− ·000005	− ·000005	+ ·000012	− ·000002	− ·000005	− ·000003	+ ·000004	− ·000006	+ ·000010	− ·000007	·000000	+ ·000001	·000000
Number of lunar days' observations in abstract	255	250	266	267	532	506	1038	214	190	203	209	413	403	816

7. Curves were now constructed to represent the variations otherwise expressed by the hourly excesses for the several seasons and phases, and a consideration of them led to the formation of the hypothesis that the bulk of the phenomenon dealt with is, properly speaking, not a lunar diurnal variation, but a solar diurnal variation that depends on the relative positions of the sun and moon, a variation such that it may be expressed, for any one season, by the formula

$$f_{c.2}(h) \cos 2\left(\frac{2\pi}{P}t\right) + f_{s.2}(h) \sin 2\left(\frac{2\pi}{P}t\right),$$

where h is the hour of the solar day, P the mean period of a lunation in mean solar days, and t the age of the moon in mean solar days; and $f_{c.2}(h), f_{s.2}(h)$, are the observed variations at new moon and when the moon has the age one-eighth of a lunation respectively. It will presently be seen that, though the typical variations $\{f(h)\}$ of one season differ from those of another, this hypothesis holds good generally in each quarter of the year separately.

Of the two characteristics of the curves that pointed in the direction of the formula, the first was the general fact that the great movements occur in them, as in curves of the mean solar diurnal variations for full lunations, in the solar *day hours*, and the night hours are relatively quiescent; or, viewed in another aspect, the significant movements occur at all hours of the lunar day in the course of a lunation, and appear earlier and earlier as the age of the moon increases. And the second characteristic was that the curves, regarded as solar diurnal curves, have generally the same form and range at intervals of half a lunation, and opposite forms at intervals of a quarter of a lunation; and this with reference separately to each of the two magnetic elements and to each season of the year.

[7*a*. It may be explained here that in the first reductions of the Bombay declination observations the categories into which the days were divided had reference (1) to the four seasons of the year, (2) to four positions of the moon in declination, and (3) to the four quarters of the moon—*not* to eight phases as in the later reductions. A short account of a first instalment of the results of these reductions, dealing with the observations of the years 1861 to 1863, was read before the Royal Society on the 2nd February, 1872; but it was the results of these earlier reductions for the full period of twenty-five years that suggested the idea of the luni-solar variation as expressed by the formula given in the last paragraph, and the computation and curving of the lunar diurnal variations of each category for that period were not completed till the 4th March, 1873. How very definite the suggestion was may be seen from the curves of figs. 50 to 53, 55 to 58, 60 to 63, and 65 to 68, which represent the lunar diurnal variations for the sixteen principal categories. To facilitate comparison, the curves for the different phases of the same season are placed under one another, the lunar time scales are arranged so that the beginning and end of the curves should correspond approximately to the same solar time, and the force scales for first quarter and last quarter are inverted in comparison with the force scales for new moon and full moon. With the curves thus arranged, the four, occupying each vertical curve-form, and of which the fifth curve is, in each case, the type (or average), can be taken into view at a single glance, and the degree of their similarity is thus easily recognised. But similarity in these curves means the same thing as the second characteristic of the curves of lunar diurnal variations, which is expressed in words in the last paragraph as follows—the curves, regarded as solar diurnal curves, have generally the same form and range at intervals of half a lunation, and opposite forms at intervals of a quarter of a lunation. The similarity is very pronounced in all seasons except the transition one—February to April—in which the inversion of character of the variations is in progress.—23rd September, 1886.]

8. It now became an object to adapt the results already obtained, which have reference to the lunar day, to the determination of the data $f_{c.2}(h)$ and $f_{s.2}(h)$ of the formula; for this purpose the following process was adopted.

As the lunar day roughly approximates to the same length as the solar day, we suppose the observed lunar diurnal variation at new moon to imply with rough approximation a solar diurnal variation of the same character; and we enter the hourly excesses in a Table having solar hours, from 0 to 23, marked at the top of the columns; under these numbers we enter the excesses of full moon, placing the number belonging to the 0th hour of lunar time under the 12th hour of solar time, and we then take the sums of the two sets of numbers. Again, we enter the excesses of first quarter, placing the number belonging to the 0th hour of lunar time under the 6th hour of solar time; and under these we enter the excesses of last quarter, placing the number of the 0th hour of lunar time under the 18th hour of solar time, and we then take the sum of these two sets of numbers; next, we subtract the latter sums from the former, and divide the results by 4, calling the series of quotients the typical variation for the quarters of the moon, that is $f_{c.2}(h)$. Similarly, substituting the hourly excesses of the phases one-eighth, three-eighths, five-eighths, and seven-eighths for those of new moon, first quarter, full moon, and last quarter respectively, we obtain the typical variation for the eighths phases, that is $f_{s.2}(h)$. In this way, of which an example will now be given, have been obtained the typical variations shown in Tables 9 and 10, using as data the numbers in Column 2 of Tables 1 to 4 for the first line of Table 9, in Column 3 of Tables 1 to 4 for the second line of Table 9, and so on.

CALCULATION of the typical Variation of Declination for the Quarters of the

	Lunar diurnal variation for—	Bombay									
		0	1	2	3	4	5	6	7	8	9
a	New Moon	+·000016	+·000018	+·000008	+·000006	−·000004	−·000008	−·000012	−·000011	−·000006	−·000003
b	Full Moon	+·000008	+·000012	+·000020	+·000010	−·000005	−·000007	+·000001	+·000006	+·000006	+·000003
c	$a+b$	+·000024	+·000030	+·000028	+·000016	−·000009	−·000015	−·000011	−·000005	·000000	·000000
d	First Quarter	−·000015	−·000015	−·000026	·000000	+·000013	+·000016	+·000018	+·000012	+·000006	−·000002
e	Last Quarter	−·000018	−·000025	−·000029	−·000014	·000000	+·000010	+·000011	+·000007	+·000005	+·000001
f	$d+e$	−·000033	−·000040	−·000055	−·000014	+·000013	+·000026	+·000029	+·000019	+·000011	−·000001
g	$c-f$	+·000057	+·000073	+·000083	+·000030	−·000022	−·000041	−·000040	−·000024	−·000011	+·000001
h	$\frac{1}{4}(c-f)=f_{c\cdot 2}(h)$	+·000014	+·000018	+·000021	+·000007	−·000005	−·000010	−·000010	−·000006	−·000003	·000000

TABLE 9.—Showing the typical Variations $f_{c.2}(h)$ and $f_{s.2}(h)$

	Bombay Astronomical Hours.	0	1	2	3	4	5	6	7	8	9
$f_{c\cdot 2}(h)$	November to January	+·000014	+·000018	+·000021	+·000007	−·000005	−·000010	−·000010	−·000006	−·000003	·000000
	February to April	+·000003	−·000001	+·000001	−·000002	·000000	−·000002	−·000005	−·000005	−·000005	−·000001
	May to July	−·000014	−·000013	−·000008	+·000001	+·000008	+·000011	+·000007	·000000	·000000	+·000001
	August to October	−·000016	−·000016	−·000004	+·000009	+·000014	+·000010	+·000004	·000000	−·000001	−·000001
$f_{s\cdot 2}(h)$	November to January	−·000011	−·000005	+·000005	+·000009	+·000017	+·000013	+·000004	−·000001	·000000	−·000002
	February to April	+·000005	+·000001	−·000008	−·000004	−·000001	+·000003	+·000003	+·000004	−·000001	−·000003
	May to July	−·000004	−·000012	−·000015	−·000023	−·000011	−·000002	+·000005	+·000004	+·000003	+·000001
	August to October	−·000004	−·000015	−·000013	−·000011	−·000007	+·000008	+·000003	+·000004	·000000	−·000001

TABLE 10.—Showing the typical Variations $f_{c.2}(h)$ and $f_{s.2}(h)$

	Bombay Astronomical Hours.	0	1	2	3	4	5	6	7	8	9
$f_{c\cdot 2}(h)$	November to January	−·000004	−·000011	−·000021	−·000023	−·000016	−·000008	−·000006	−·000001	−·000001	+·000004
	February to April	+·000001	−·000003	−·000006	−·000013	−·000010	−·000007	−·000001	−·000003	−·000003	+·000001
	May to July	+·000001	−·000002	−·000009	−·000010	−·000008	−·000003	+·000002	+·000003	+·000002	·000000
	August to October	−·000004	−·000005	−·000003	−·000008	−·000006	−·000001	−·000003	−·000005	−·000001	·000000
$f_{s\cdot 2}(h)$	November to January	+·000031	+·000024	+·000014	+·000004	·000000	−·000011	−·000016	−·000009	−·000011	−·000011
	February to April	+·000021	+·000020	+·000015	+·000006	+·000001	−·000001	·000000	−·000003	−·000007	−·000011
	May to July	+·000011	+·000004	+·000010	+·000009	+·000002	·000000	−·000004	−·000005	−·000003	−·000001
	August to October	+·000017	+·000013	+·000008	+·000006	−·000002	−·000010	−·000008	−·000005	−·000007	−·000005

The variations of Tables 9 and

Moon in the Quarter November to January—that is, of the Variation $f_{c.2}(h)$.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
−·000001	+·000004	+·000004	+·000006	+·000004	+·000005	+·000001	−·000004	−·000016	−·000009	−·000001	−·000008	−·000006	+·000008
+·000002	+·000004	+·000008	+·000004	+·000001	−·000006	−·000007	−·000009	−·000015	−·000011	−·000018	−·000017	+·000001	+·000009
+·000001	+·000008	+·000012	+·000010	+·000005	−·000001	−·000006	−·000013	−·000031	−·000020	−·000019	−·000025	−·000005	+·000017
+·000002	−·000001	−·000006	−·000002	−·000006	−·000009	−·000005	+·000004	+·000006	+·000006	+·000006	+·000008	−·000002	−·000004
+·000003	·000000	−·000003	·000000	+·000001	+·000006	+·000010	+·000011	+·000010	+·000009	+·000011	+·000014	·000000	−·000011
+·000005	−·000001	−·000009	−·000002	−·000005	−·000003	+·000005	+·000015	+·000016	+·000015	+·000017	+·000022	−·000002	−·000015
−·000004	+·000009	+·000021	+·000012	+·000010	+·000002	−·000011	−·000028	−·000047	−·000035	−·000036	−·000047	−·000003	+·000032
−·000001	+·000002	+·000005	+·000003	+·000002	·000000	−·000003	−·000007	−·000012	−·000009	−·000009	−·000012	−·000001	+·000008

of Declination at Bombay for each Quarter of the Year.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
−·000001	+·000002	+·000005	+·000003	+·000002	·000000	−·000003	−·000007	−·000012	−·000009	−·000009	−·000012	−·000001	+·000008
−·000002	·000000	+·000003	+·000001	+·000002	+·000001	−·000002	−·000004	−·000003	+·000004	+·000002	+·000006	+·000008	+·000001
+·000001	+·000001	−·000001	·000000	+·000001	·000000	+·000001	+·000007	+·000009	+·000012	+·000005	−·000006	−·000014	−·000011
−·000002	+·000002	+·000001	+·000003	+·000004	+·000004	+·000004	+·000007	+·000007	+·000007	+·000003	−·000007	−·000015	−·000019
−·000003	−·000002	·000000	+·000001	+·000004	+·000003	+·000006	+·000007	+·000006	+·000001	−·000006	−·000014	−·000016	−·000011
−·000003	−·000003	+·000001	+·000001	+·000006	+·000005	+·000004	·000000	−·000002	−·000005	·000000	−·000006	·000000	−·000002
·000000	·000000	+·000002	+·000003	+·000002	·000000	−·000001	−·000004	+·000001	+·000010	+·000011	+·000012	+·000011	+·000005
+·000001	−·000001	−·000004	−·000002	+·000001	+·000002	+·000002	+·000002	+·000004	+·000007	+·000017	+·000009	+·000003	−·000002

of Horizontal Force at Bombay for each Quarter of the Year.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
+·000001	+·000003	−·000002	·000000	−·000003	−·000004	−·000002	+·000001	+·000004	+·000007	+·000019	+·000027	+·000024	+·000013
+·000004	+·000004	·000000	−·000004	−·000003	−·000004	−·000003	−·000001	+·000004	+·000006	+·000009	+·000012	+·000009	+·000009
−·000003	+·000001	·000000	+·000002	+·000002	+·000002	+·000004	+·000003	+·000003	+·000006	+·000003	+·000002	−·000002	+·000001
−·000001	+·000002	+·000002	+·000001	·000000	−·000003	−·000003	+·000003	+·000005	+·000003	+·000004	+·000014	+·000002	·000000
−·000014	−·000002	−·000004	−·000001	−·000003	−·000003	−·000007	−·000012	−·000013	−·000010	−·000008	+·000011	+·000022	+·000032
−·000013	−·000006	−·000005	−·000003	−·000006	−·000011	−·000009	−·000008	−·000008	−·000008	−·000002	+·000001	+·000015	+·000024
−·000001	−·000004	−·000002	−·000001	+·000001	−·000005	−·000004	−·000009	−·000006	−·000005	−·000001	+·000003	+·000006	+·000009
−·000009	−·000003	+·000001	+·000001	−·000006	−·000002	−·000003	−·000005	−·000004	−·000005	+·000002	+·000001	+·000006	+·000012

10 are curved in figs. 1 to 16.

9. Considering now the typical variation curves (figs. 1 to 16), we see that those which are the most definite and systematic in character, and have the largest range, whether of declination or horizontal force, are for the winter season; and, in the case of each type, the curves for the other seasons are definitely related to these. Thus, as to declination, between winter and summer there is an inversion of the types of both figs. 1 and 5; and the type-curves of the transition period, the spring season, have but a small range, and in them the prominent features of the winter curves are all but obliterated. Necessarily there is also a return inversion of the type in another season, and it occurs between mid-autumn and mid-winter. On the other hand, the horizontal force types (figs. 9 to 16), although of largest range in winter, are never reversed in character, but contract only to a minimum range, which is reached in the summer season.

The typical diurnal variation for November to January of horizontal force at new moon (fig. 9), and that of declination at the one-eighth phase reversed to correspond to the seven-eighths phase (fig. 5 reversed), have generally a maximum about three hours before noon, a minimum about three hours after noon, and are nearly *nil* for three or four hours before and after midnight; those of declination at new moon (fig. 1) and of horizontal force at the one-eighth phase (fig. 13) have a principal upward inflexion with a turning point about noon, two nearly equal downward inflexions with turning points about five or six hours before and after noon, and little or no departure from the mean line for three or four hours before and after midnight. For the summer and autumn seasons similar descriptions of the typical diurnal variations would apply, providing due allowance were made for the double reversal in the course of the year of the declination types.

Comparing figs. 3 and 4 with 1, and figs. 7 and 8 with 5, all declination curves, it may be noted, as a minor characteristic, that the principal inflexions occur generally somewhat earlier in summer and autumn than in winter.

10. The general similarity of character of figs. 1 and 13, representing the $f_{c.2}(h)$ and $f_{s.2}(h)$ data of our formula for the declination and horizontal force respectively, and the opposition of character of figs. 5 and 9, representing conversely the $f_{s.2}(h)$ and $f_{c.2}(h)$ data for the elements taken in the same order, call for further consideration. These relations may be translated into a statement that, in the winter season, the luni-solar variation of declination due to new moon is similar to the luni-solar variation of horizontal force due to the one-eighth phase; and the luni-solar variation of horizontal force due to new moon is similar to the luni-solar variation of declination due to the seven-eighths phase. In other words, the luni-solar variation of declination due to any phase of the moon is similar to the luni-solar variation of horizontal force due to a phase later by one-eighth of a lunation. In the summer and autumn seasons it is, on the other hand, the luni-solar variations of horizontal force that precede by one-eighth of a lunation the similar variations of declination; this will be seen by comparing figs. 11 and 12 with figs. 7 and 8, and figs. 15 and 16 with figs. 3 and 4 reversed.

The fact of harmonious relations of this kind being found to subsist between results derived from long series of observations of two *independent* instruments, we cannot but regard as strong testimony to the reality of the phenomena now brought to light; neither can we refrain from claiming for such results a modest place amongst the phenomenal laws of terrestrial magnetism that must ultimately stand in the same relation to a physical theory of terrestrial magnetism that KEPLER'S laws stand in towards the theory of gravitation.

11. At this stage the question was put—With what approach to completeness does the typical variation in each case represent the four actual variations from which it was derived, or how much of a mean lunar variation is there in these over and above the typical variation? Subtracting (say) $f_{c.2}(h)$ from the observed variation at new moon and full moon, and adding it to the observed variation at first quarter and last quarter in such a way that the 0th, 6th, 12th, and 18th hours of $f_{c.2}(h)$ are compared with the 0th hour of new moon, first quarter, full moon, and last quarter respectively; the 1st, 7th, 13th, and 19th hours of $f_{c.2}(h)$ with the 1st hour of new moon, first quarter, &c., respectively, and so on, we obtain four sets of residual variations, each commencing with the 0th hour of the lunar day; and, taking the mean of these four, we obtain the residual lunar diurnal variation that is left after appropriately eliminating the typical variation for the four quarters. A similar procedure, using the observed variations at the eighths phases, gives corresponding residual variations for the eighths phases.

12. On curving these residuals for each quarter of the year—eight in all for each magnetic element—and comparing each curve with the corresponding typical variation curve, they were all found to be of small relative range, but most of them had a definite character, in which the principal harmonic element was that which has the lunar day for its period. In the latter fact we found a suggestion that, although our formula disposes of the bulk of the phenomena for which an expression is to be found, the addition to it of two more independent terms would not only make it mathematically more complete, but would render it further expressive of an otherwise neglected, but significant, element in the luni-solar variation. Making this addition, the formula becomes

$$f_{c.1}(h)\cos\left(\frac{2\pi}{P}t\right)+f_{s.1}(h)\sin\left(\frac{2\pi}{P}t\right)+f_{c.2}(h)\cos 2\left(\frac{2\pi}{P}t\right)+f_{s.2}(h)\sin 2\left(\frac{2\pi}{P}t\right),*$$

* If ϕ be written for the angle $\frac{2\pi}{P}t$, and θ for the angle $\frac{2\pi}{24}h$, the formula may easily be transformed into

$$F_{c.1}(\phi)\cos\left(\frac{2\pi}{24}h\right)+F_{s.1}(\phi)\sin\left(\frac{2\pi}{24}h\right)+F_{c.2}(\phi)\cos 2\left(\frac{2\pi}{24}h\right)+F_{s.2}(\phi)\sin 2\left(\frac{2\pi}{24}h\right),$$

in which $F_{c.1}(\phi)$, $F_{s.1}(\phi)$ are variations, of constant types, having the period of a lunation, and $F_{c.2}(\phi)$, $F_{s.2}(\phi)$ are variations, of constant types, having a period of half a lunation; and all these swell and contract with a wave-like oscillation—$F_{c.1}(\phi)$, $F_{s.1}(\phi)$ in the period of a day, and $F_{c.2}(\phi)$, $F_{s.2}(\phi)$ in the each term of which is symbolical of a definite physical conception, viz., that an otherwise constant variation swells and contracts with a wave-like motion, as the age of the moon increases, between the limits $-f(h)$ and $+f(h)$.

13. If the initial new moon occur at h' hours of the initial solar day, from the beginning of which time is reckoned in solar astronomical hours, the age of the moon will become $h-h'$, the period of a lunation 24P; and for $\frac{2\pi}{P}t$ may be substituted $\frac{2\pi}{24P}(h-h')$: and if, further, $f(h)$ be expressed in the form

$$a_1 \cos\frac{2\pi}{24}h + b_1 \sin\frac{2\pi}{24}h + a_2 \cos 2\left(\frac{2\pi}{24}h\right) + b_2 \sin 2\left(\frac{2\pi}{24}h\right) + \&c.,$$

our extended formula may easily be transformed into

$$\begin{aligned}
&A_1 \cos\left\{\frac{2\pi}{24}h\left(1-\frac{1}{P}\right)+\alpha\right\}+B_1 \cos\left\{\frac{2\pi}{24}2h\left(1-\frac{1}{2P}\right)+\beta\right\}\\
&+C_1 \cos\left\{\frac{2\pi}{24}2h\left(1+\frac{1}{2P}\right)+\gamma\right\}+D_1 \cos\left\{\frac{2\pi}{24}h\left(1+\frac{1}{P}\right)+\delta\right\}\\
&+A_2 \cos\left\{\frac{2\pi}{24}h\left(1-\frac{2}{P}\right)+\epsilon\right\}+B_2 \cos\left\{\frac{2\pi}{24}2h\left(1-\frac{1}{P}\right)+\zeta\right\}\\
&+C_2 \cos\left\{\frac{2\pi}{24}2h\left(1+\frac{1}{P}\right)+\eta\right\}+D_2 \cos\left\{\frac{2\pi}{24}h\left(1+\frac{2}{P}\right)+\theta\right\},
\end{aligned}$$

where the numbers A_1, B_1, C_1, D_1, &c., and the angles α, β, γ, δ, &c., are constants; that is to say, it may—inclusive of the first four terms of $f(h)$—be transformed into eight simple waves whose periods, in solar hours, are

$$24\frac{P}{P-1};\ 12\frac{2P}{2P-1};\ 12\frac{2P}{2P+1};\ 24\frac{P}{P+1};\ 24\frac{P}{P-2};\ 12\frac{P}{P-1};\ 12\frac{P}{P+1};\ \text{and}\ 24\frac{P}{P+2}.$$

Of these periods the first and sixth are the lunar day and half-day respectively; from which it follows that, even if our extended formula be a substantially correct expres-

period of half-a-day. It thus appears that θ and ϕ are reciprocally related, so that the period of either may be regarded as that of the variation of constant type, and the period of the other is then that in which the variation of constant type oscillates, whilst the complex variation of the formula remains identically the same; and this result is general so long as the number of terms in the formula is the same as the number of terms in each of the variations of constant type. The name—the luni-solar variation—has been chosen to distinguish the variation expressed by the formula as one involving the periods of a lunation and the solar day, and of sub-multiples of these periods; and the name—typical variation—has been given to the variations of constant type.

sion of the phenomena in question, there is nothing anomalous in our finding, as we do in fact, definite lunar diurnal variations from the observations of *complete* lunations. It will, indeed, be seen further on (paragraph 17) that the lunar half-day wave is one of the two most prominent, and that the lunar day wave has, generally speaking, an amplitude as great as any of the remaining five.

14. Proceeding now to find the values of $f_{c.1}(h), f_{s.1}(h)$, having entered the observed lunar diurnal variation for new moon in a Table having the solar hours, from 0 to 23, marked at the top of the columns, commencing with 0 hours of the lunar day, we enter underneath this the observed lunar diurnal variation for full moon, commencing with 12 hours, and then subtract the lower entries from the upper: half the difference we take to be the value of $f_{c.1}(h)$ as derived from the new moon and full moon variations. Similarly, entering the observed lunar diurnal variation for first quarter, commencing with 18 hours, and under it the observed lunar diurnal variation for last quarter, commencing with 6 hours, and subtracting the lower numbers and dividing by 2, we obtain the value of $f_{s.1}(h)$ as derived from the first quarter and last quarter variations. In like manner we obtain from the observed lunar diurnal variation for the one-eighth and five-eighths phases the value of $f_{c.1}(h)\cos 45^\circ + f_{s.1}(h)\sin 45^\circ$ or $\frac{1}{\sqrt{2}}\{f_{c.1}(h)+f_{s.1}(h)\}=a$ (say); and from the variations for the three-eighths and seven-eighths phases the value of $\frac{1}{\sqrt{2}}\{-f_{c.1}(h)+f_{s.1}(h)\}=b$ (say): combining the last two quantities, we find $\frac{1}{\sqrt{2}}(a-b)=f_{c.1}$ and $\frac{1}{\sqrt{2}}(a+b)=f_{s.1}$ as values derived from the variations of the eighths phases. In Tables 11 to 18 are collected together the results of these various calculations.

TABLE 11.—Declination.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay									
			0	1	2	3	4	5	6	7	8	9
a	New and full moon	$f_{c\cdot 1}(h)$	+·000004	+·000003	−·000006	−·000002	·000000	·000000	−·000006	−·000008	−·000006	−·000003
b	First and last quarters	$f_{s\cdot 1}(h)$	+·000001	+·000005	+·000001	+·000007	+·000006	+·000003	+·000003	+·000002	·000000	−·000001
c	One and five-eighths	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot 1}(h)+f_{s\cdot 1}(h)\right\}$	−·000001	+·000005	−·000004	−·000011	−·000001	−·000001	+·000001	−·000003	−·000006	−·000007
d	Three and seven-eighths	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot 1}(h)+f_{s\cdot 1}(h)\right\}$	+·000012	+·000005	−·000003	+·000004	−·000003	·000000	+·000001	−·000001	−·000002	+·000001
e	Eighths	$f_{c\cdot 1}(h)$	·000000	+·000004	−·000003	−·000008	·000000	−·000001	·000000	−·000002	−·000004	−·000005
f	Eighths	$f_{s\cdot 1}(h)$	+·000008	+·000004	−·000002	+·000003	−·000002	·000000	·000000	·000000	−·000002	·000000
g	Mean of a and e	$f_{c\cdot 1}(h)$	+·000002	+·000003	−·000004	−·000005	·000000	−·000001	−·000003	−·000005	−·000005	−·000004
h	Mean of b and f	$f_{s\cdot 1}(h)$	+·000005	+·000004	·000000	+·000005	+·000002	+·000002	+·000002	+·000001	−·000001	·000000

TABLE 12.—Declination.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay									
			0	1	2	3	4	5	6	7	8	9
a	New and full moon	$f_{c\cdot 1}(h)$	·000000	·000000	−·000001	−·000003	−·000003	−·000002	−·000001	−·000004	−·000004	−·000002
b	First and last quarters	$f_{s\cdot 1}(h)$	−·000003	−·000001	+·000001	+·000001	·000000	+·000001	·000000	·000000	·000000	·000000
c	One and five-eighths	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot 1}(h)+f_{s\cdot 1}(h)\right\}$	−·000001	−·000003	−·000008	−·000001	−·000001	−·000003	·000000	+·000004	+·000002	+·000002
d	Three and seven-eighths	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot 1}(h)+f_{s\cdot 1}(h)\right\}$	−·000005	−·000001	−·000002	+·000005	+·000004	+·000004	+·000005	−·000005	−·000004	−·000003
e	Eighths	$f_{c\cdot 1}(h)$	−·000001	−·000002	−·000006	−·000001	·000000	−·000002	·000000	+·000003	+·000001	+·000002
f	Eighths	$f_{s\cdot 1}(h)$	−·000004	−·000001	−·000002	+·000004	+·000003	+·000003	+·000003	−·000004	−·000003	−·000002
g	Mean of a and e	$f_{c\cdot 1}(h)$	−·000001	−·000001	−·000004	−·000002	−·000002	−·000002	·000000	−·000001	−·000001	·000000
h	Mean of b and f	$f_{s\cdot 1}(h)$	−·000003	−·000001	·000000	+·000002	+·000001	+·000002	+·000001	−·000002	−·000001	−·000001

TABLE 13.—Declination.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay									
			0	1	2	3	4	5	6	7	8	9
a	New and full moon	$f_{c\cdot 1}(h)$	+·000004	−·000001	−·000003	−·000005	−·000005	+·000004	+·000006	+·000003	+·000001	−·000001
b	First and last quarters	$f_{s\cdot 1}(h)$	+·000001	·000000	+·000001	+·000001	+·000003	+·000001	−·000003	−·000002	·000000	+·000002
c	One and five-eighths	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot 1}(h)+f_{s\cdot 1}(h)\right\}$	+·000004	−·000008	−·000010	−·000014	−·000008	−·000005	−·000002	−·000005	−·000001	−·000003
d	Three and seven-eighths	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot 1}(h)+f_{s\cdot 1}(h)\right\}$	−·000001	−·000001	+·000013	+·000005	+·000003	·000000	+·000002	−·000001	−·000001	−·000003
e	Eighths	$f_{c\cdot 1}(h)$	+·000003	−·000005	−·000007	−·000010	−·000006	−·000003	−·000001	−·000004	·000000	−·000002
f	Eighths	$f_{s\cdot 1}(h)$	−·000001	·000000	+·000009	+·000004	+·000002	·000000	+·000001	−·000001	·000000	−·000002
g	Mean of a and e	$f_{c\cdot 1}(h)$	+·000003	−·000003	−·000005	−·000008	−·000006	·000000	+·000002	·000000	·000000	−·000002
h	Mean of b and f	$f_{s\cdot 1}(h)$	·000000	·000000	+·000005	+·000002	+·000003	·000000	−·000001	−·000001	·000000	·000000

November to January.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
−·000001	·000000	−·000002	+·000001	+·000001	+·000005	+·000004	+·000002	·000000	+·000001	+·000008	+·000004	−·000003	·000000
·000000	·000000	−·000001	−·000001	−·000003	−·000007	−·000007	−·000003	−·000002	−·000001	−·000002	−·000003	−·000001	+·000003
−·000003	+·000002	−·000001	·000000	+·000004	+·000008	+·000005	+·000002	−·000001	+·000001	+·000002	+·000010	+·000001	+·000005
·000000	·000000	−·000003	+·000001	−·000004	−·000005	−·000007	−·000004	−·000004	−·000001	−·000001	+·000006	+·000006	+·000001
−·000002	+·000002	−·000001	·000000	+·000003	+·000005	+·000004	+·000002	·000000	·000000	+·000002	+·000007	·000000	+·000004
·000000	·000000	−·000002	+·000001	−·000003	−·000003	−·000005	−·000003	−·000003	·000000	−·000001	+·000004	+·000004	+·000001
−·000002	+·000001	−·000001	·000000	+·000002	+·000005	+·000004	+·000002	·000000	+·000001	+·000005	+·000006	−·000001	+·000002
·000000	·000000	−·000002	·000000	−·000003	−·000005	−·000006	−·000003	−·000002	−·000001	−·000002	·000000	+·000001	+·000002

February to April.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
+·000002	+·000002	+·000007	+·000003	+·000006	+·000009	+·000003	+·000005	+·000002	−·000001	−·000003	−·000002	−·000002	−·000005
−·000004	−·000005	·000000	+·000002	·000000	−·000002	·000000	+·000004	+·000004	+·000001	+·000002	+·000002	+·000006	−·000012
+·000004	+·000004	+·000001	+·000009	+·000007	+·000002	−·000003	−·000003	·000000	−·000005	−·000003	+·000002	−·000002	−·000006
+·000004	−·000006	−·000003	−·000006	−·000005	·000000	−·000005	+·000001	+·000006	+·000007	+·000006	−·000005	+·000003	+·000003
+·000003	+·000003	+·000001	+·000006	+·000005	+·000001	−·000002	−·000002	·000000	−·000003	−·000002	+·000001	−·000002	−·000004
+·000003	−·000004	−·000002	−·000004	−·000004	·000000	−·000003	·000000	+·000004	+·000005	+·000004	−·000003	+·000002	+·000002
+·000003	+·000002	+·000004	+·000005	+·000006	+·000005	+·000001	+·000002	+·000001	−·000002	−·000003	·000000	−·000002	−·000004
·000000	−·000004	−·000001	−·000001	−·000002	−·000001	−·000002	+·000002	+·000004	+·000003	+·000003	−·000001	+·000004	−·000005

May to July.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
+·000001	−·000001	+·000003	−·000001	−·000001	+·000001	+·000002	+·000001	+·000003	·000000	−·000006	−·000007	+·000004	−·000001
·000000	·000000	−·000001	−·000002	−·000004	−·000006	−·000005	−·000002	−·000002	+·000002	·000000	−·000003	+·000005	+·000006
·000000	+·000006	·000000	+·000001	+·000003	+·000006	+·000002	+·000007	+·000008	−·000003	+·000002	+·000010	+·000001	+·000008
−·000001	−·000006	−·000007	−·000006	−·000006	−·000002	−·000005	−·000005	−·000001	+·000004	−·000003	+·000012	+·000004	+·000002
·000000	+·000004	·000000	·000000	+·000002	+·000004	+·000001	+·000005	+·000005	−·000002	+·000001	+·000007	·000000	+·000006
−·000001	−·000004	−·000005	−·000004	−·000004	−·000001	−·000003	−·000003	·000000	+·000003	−·000002	+·000008	+·000003	+·000002
+·000001	+·000001	+·000002	·000000	·000000	+·000003	+·000002	+·000008	+·000004	−·000001	−·000002	·000000	+·000002	+·000002
−·000001	−·000002	−·000003	−·000003	−·000004	−·000004	−·000004	−·000003	−·000001	+·000003	−·000001	+·000003	+·000004	+·000004

TABLE 14.—Declination.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay									
			0	1	2	3	4	5	6	7	8	9
a	New and full moon . .	$f_{c\cdot1}(h)$	−·000003	−·000001	−·000003	+·000001	+·000002	·000000	·000000	+·000005	+·000006	+·000001
b	First and last quarters .	$f_{s\cdot1}(h)$	−·000001	−·000015	−·000002	·000000	·000000	+·000004	−·000001	·000000	−·000002	·000000
c	One and five-eighths . .	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$.	+·000003	+·000009	−·000003	+·000008	·000000	+·000004	+·000004	·000000	+·000001	+·000002
d	Three and seven-eighths.	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$	−·000004	−·000002	−·000003	−·000003	+·000004	+·000009	+·000007	+·000003	+·000003	−·000002
e	Eighths	$f_{c\cdot1}(h)$	+·000002	+·000006	−·000002	+·000005	·000000	+·000003	+·000003	·000000	+·000001	+·000002
f	Eighths	$f_{s\cdot1}(h)$	−·000003	−·000002	−·000002	−·000002	−·000003	+·000006	+·000005	+·000002	+·000002	−·000002
g	Mean of a and e . . .	$f_{c\cdot1}(h)$	·000000	+·000002	−·000003	+·000003	+·000001	+·000001	+·000002	+·000003	+·000003	+·000002
h	Mean of b and f . . .	$f_{s\cdot1}(h)$	−·000002	−·000009	−·000002	−·000001	−·000001	+·000005	+·000002	+·000001	·000000	−·000001

TABLE 15.—Horizontal Force.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay									
			0	1	2	3	4	5	6	7	8	9
a	New and full moon . .	$f_{c\cdot1}(h)$	·000000	−·000004	−·000004	−·000003	−·000003	−·000003	·000000	·000000	·000000	+·000004
b	First and last quarters .	$f_{s\cdot1}(h)$	−·000014	−·000021	−·000009	−·000008	−·000005	·000000	·000000	+·000003	+·000005	+·000003
c	One and five-eighths . .	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$.	−·000014	−·000004	+·000006	+·000003	−·000009	+·000009	+·000011	+·000015	+·000009	+·000011
d	Three and seven-eighths.	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$	·000000	·000000	−·000008	−·000004	−·000007	−·000002	−·000003	+·000005	+·000006	−·000001
e	Eighths	$f_{c\cdot1}(h)$	−·000010	−·000003	+·000004	+·000002	−·000006	+·000006	+·000008	+·000011	+·000006	+·000008
f	Eighths	$f_{s\cdot1}(h)$	·000000	·000000	−·000005	−·000003	−·000005	−·000001	−·000002	+·000004	+·000004	−·000001
g	Mean of a and e . . .	$f_{c\cdot1}(h)$	−·000005	−·000003	·000000	·000000	−·000005	+·000001	+·000004	+·000006	+·000003	+·000006
h	Mean of b and f . . .	$f_{s\cdot1}(h)$	−·000007	−·000010	−·000007	−·000005	−·000005	−·000001	−·000001	+·000003	+·000004	+·000001

TABLE 16.—Horizontal Force.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay									
			0	1	2	3	4	5	6	7	8	9
a	New and full moon . .	$f_{c\cdot1}(h)$	−·000004	−·000002	+·000001	−·000004	−·000007	−·000004	+·000001	+·000003	+·000005	−·000001
b	First and last quarters .	$f_{s\cdot1}(h)$	−·000004	·000000	+·000007	+·000014	+·000010	+·000006	+·000005	+·000004	·000000	·000000
c	One and five-eighths . .	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$.	+·000011	−·000008	−·000019	−·000012	·000000	+·000006	·000000	−·000008	−·000008	+·000003
d	Three and seven-eighths.	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$	+·000007	+·000007	+·000008	−·000008	−·000012	−·000008	−·000004	−·000004	+·000005	−·000010
e	Eighths	$f_{c\cdot1}(h)$	+·000008	−·000005	−·000013	−·000008	·000000	+·000004	·000000	−·000006	−·000005	+·000002
f	Eighths	$f_{s\cdot1}(h)$	+·000005	+·000005	+·000005	−·000005	−·000008	−·000006	−·000003	−·000003	+·000003	−·000007
g	Mean of a and e . . .	$f_{c\cdot1}(h)$	+·000002	−·000004	−·000006	−·000006	−·000003	·000000	·000000	−·000001	·000000	·000000
h	Mean of b and f . . .	$f_{s\cdot1}(h)$	·000000	+·000003	+·000006	+·000004	+·000001	·000000	+·000001	+·000001	+·000001	−·000004

August to October.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
+·000002	·000000	–·000002	·000000	–·000001	–·000002	+·000001	·000000	–·000002	·000000	·000000	–·000005	–·000006	+·000001
+·000003	–·000001	–·000001	·000000	–·000003	·000000	·000000	–·000001	·000000	+·000003	+·000006	+·000007	+·000009	·000000
–·000002	–·000004	·000000	·000000	–·000001	–·000002	–·000003	–·000005	·000000	–·000002	–·000007	–·000005	–·000002	–·000004
–·000001	+·000002	–·000001	·000000	–·000003	+·000002	+·000003	+·000002	–·000003	–·000002	+·000001	+·000002	–·000005	·000000
–·000001	–·000003	·000000	·000000	–·000001	–·000001	–·000002	–·000003	·000000	–·000001	–·000005	–·000004	–·000002	–·000003
·000000	+·000002	–·000001	·000000	–·000002	+·000001	+·000002	+·000001	–·000002	–·000001	+·000001	+·000002	–·000004	·000000
·000000	–·000001	–·000001	·000000	–·000001	–·000002	·000000	–·000002	–·000001	–·000001	–·000002	–·000004	–·000004	–·000001
+·000001	·000000	–·000001	·000000	–·000003	·000000	+·000001	·000000	–·000001	+·000001	+·000004	+·000005	+·000003	·000000

November to January.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
·000000	–·000004	–·000003	–·000005	·000000	+·000002	+·000001	+·000002	+·000005	+·000007	+·000002	+·000003	–·000001	+·000004
+·000009	+·000010	·000000	+·000005	+·000003	+·000005	+·000008	+·000013	+·000007	+·000001	+·000001	–·000003	·000000	–·000007
+·000008	+·000004	–·000002	+·000005	+·000004	–·000006	–·000009	–·000007	+·000002	–·000003	–·000012	–·000005	–·000009	–·000016
–·000001	·000000	+·000011	–·000003	·000000	+·000005	+·000004	+·000006	+·000006	+·000008	–·000003	–·000001	–·000008	+·000004
+·000006	+·000003	–·000001	+·000003	+·000003	–·000004	–·000006	–·000005	+·000002	–·000002	–·000008	–·000004	–·000006	–·000011
–·000001	·000000	+·000008	–·000002	·000000	+·000003	+·000003	+·000004	+·000004	+·000005	–·000002	–·000001	–·000005	+·000003
+·000003	·000000	–·000002	–·000001	+·000001	–·000001	–·000003	–·000001	+·000004	+·000002	–·000003	·000000	–·000004	–·000004
+·000004	+·000005	+·000004	+·000002	+·000002	+·000004	+·000005	+·000008	+·000006	+·000003	·000000	–·000002	–·000002	–·000002

February to April.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
+·000003	·000000	+·000006	+·000005	+·000003	+·000003	–·000003	·000000	–·000001	–·000001	+·000001	+·000001	–·000002	+·000003
–·000002	–·000001	–·000001	–·000005	–·000007	–·000004	+·000001	+·000004	+·000002	–·000004	–·000003	–·000014	–·000009	–·000006
+·000013	+·000015	+·000018	+·000003	–·000001	+·000009	+·000003	–·000001	·000000	–·000003	–·000009	–·000003	–·000014	+·000001
–·000011	–·000008	–·000004	+·000002	+·000002	+·000007	+·000003	+·000008	·000000	+·000003	+·000006	+·000008	+·000002	+·000005
+·000009	+·000011	+·000013	+·000002	–·000001	+·000006	+·000002	·000000	·000000	–·000002	–·000006	–·000002	–·000010	·000000
–·000008	–·000006	–·000003	+·000002	+·000002	+·000005	+·000002	+·000005	·000000	+·000002	+·000004	+·000005	+·000001	+·000003
+·000006	+·000006	+·000009	+·000004	+·000001	+·000005	·000000	·000000	–·000001	–·000002	–·000003	·000000	–·000006	+·000002
–·000005	–·000004	–·000002	–·000002	–·000002	·000000	+·000002	+·000005	+·000001	–·000001	·000000	–·000004	–·000004	–·000001

TABLE 17.—Horizontal Force.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay 0	1	2	3	4	5	6	7	8	9
a	New and full moon	$f_{c\cdot1}(h)$	−·000007	−·000006	−·000001	−·000003	−·000003	·000000	+·000003	+·000002	·000000	+·000003
b	First and last quarters	$f_{s\cdot1}(h)$	·000000	+·000005	+·000001	+·000005	+·000009	·000000	·000000	+·000002	−·000006	−·000004
c	One and five-eighths	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$	+·000004	+·000003	+·000009	+·000005	+·000004	+·000004	−·000004	·000000	−·000005	−·000001
d	Three and seven-eighths	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$	+·000021	+·000008	+·000008	+·000016	+·000008	+·000006	+·000002	·000000	·000000	−·000008
e	Eighths	$f_{c\cdot1}(h)$	+·000003	+·000002	+·000006	+·000004	+·000003	+·000003	−·000003	·000000	−·000003	·000000
f	Eighths	$f_{s\cdot1}(h)$	+·000015	+·000006	+·000006	+·000011	+·000006	+·000004	+·000001	·000000	·000000	−·000005
g	Mean of a and e	$f_{c\cdot1}(h)$	−·000002	−·000002	+·000003	·000000	·000000	+·000002	·000000	+·000001	−·000002	+·000001
h	Mean of b and f	$f_{s\cdot1}(h)$	+·000008	+·000005	+·000003	+·000008	+·000007	+·000002	+·000001	+·000001	−·000003	−·000005

TABLE 18.—Horizontal Force.

	Derived from lunar diurnal variations for what phases.	Typical variation.	Bombay 0	1	2	3	4	5	6	7	8	9
a	New and full moon	$f_{c\cdot1}(h)$	−·000002	·000000	·000000	−·000004	−·000004	+·000001	·000000	−·000004	−·000002	+·000003
b	First and last quarters	$f_{s\cdot1}(h)$	+·000012	+·000012	·000000	+·000003	+·000012	+·000001	−·000002	+·000001	−·000003	−·000006
c	One and five-eighths	$\frac{1}{\sqrt{2}}\left\{f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$	−·000001	−·000004	+·000005	−·000016	−·000015	−·000014	·000000	+·000002	+·000005	−·000003
d	Three and seven-eighths	$\frac{1}{\sqrt{2}}\left\{-f_{c\cdot1}(h)+f_{s\cdot1}(h)\right\}$	−·000005	+·000001	+·000013	−·000005	−·000013	−·000006	−·000012	−·000006	−·000008	−·000008
e	Eighths	$f_{c\cdot1}(h)$	·000000	−·000003	+·000003	−·000011	−·000011	−·000010	·000000	+·000001	+·000004	−·000002
f	Eighths	$f_{s\cdot1}(h)$	−·000003	+·000001	+·000009	−·000004	−·000009	−·000004	−·000008	−·000004	−·000006	−·000005
g	Mean of a and e	$f_{c\cdot1}(h)$	−·000001	−·000001	+·000002	−·000008	−·000008	−·000004	·000000	−·000001	+·000001	·000000
h	Mean of b and f	$f_{s\cdot1}(h)$	+·000004	+·000007	+·000004	·000000	+·000001	−·000001	−·000005	−·000001	−·000004	−·000006

May to July.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
−·000002	+·000001	+·000001	+·000003	+·000004	+·000004	+·000002	+·000005	+·000005	+·000003	−·000001	−·000004	−·000005	−·000002
−·000003	−·000001	+·000004	+·000002	−·000001	+·000001	+·000002	+·000001	+·000005	+·000002	−·000004	−·000013	·000000	−·000010
+·000002	−·000002	+·000003	+·000004	+·000005	·000000	+·000007	+·000006	·000000	·000000	−·000007	−·000007	−·000013	−·000006
−·000004	−·000004	−·000006	−·000008	·000000	−·000008	−·000007	−·000017	−·000016	−·000008	−·000010	+·000003	+·000007	+·000011
+·000001	−·000002	+·000002	+·000003	+·000004	·000000	+·000005	+·000004	·000000	·000000	−·000005	−·000005	−·000009	−·000004
−·000003	−·000003	−·000004	−·000005	·000000	−·000005	−·000005	−·000012	−·000011	−·000006	−·000007	+·000002	+·000005	+·000008
·000000	·000000	+·000002	+·000003	+·000004	+·000002	+·000004	+·000005	+·000003	+·000001	−·000003	−·000005	−·000007	−·000003
−·000003	−·000002	·000000	−·000002	−·000001	−·000002	−·000001	−·000005	−·000003	−·000002	−·000006	−·000005	+·000002	−·000001

August to October.

Astronomical Hours.

10	11	12	13	14	15	16	17	18	19	20	21	22	23
·000000	−·000004	+·000005	·000000	+·000006	+·000003	+·000001	+·000001	+·000005	+·000002	−·000002	−·000001	−·000005	−·000005
−·000007	−·000008	−·000009	−·000009	+·000001	−·000003	−·000005	−·000009	−·000002	+·000001	−·000003	+·000005	+·000007	+·000009
+·000007	+·000010	+·000008	+·000007	+·000006	+·000002	·000000	+·000002	+·000001	·000000	−·000007	+·000005	+·000005	−·000001
+·000004	·000000	+·000007	+·000001	+·000001	+·000007	+·000005	+·000007	+·000008	+·000008	+·000007	+·000006	−·000008	−·000007
+·000005	+·000007	+·000006	+·000005	+·000004	+·000001	·000000	+·000001	·000000	·000000	−·000005	+·000004	+·000004	−·000001
+·000003	·000000	+·000005	·000000	+·000001	+·000005	+·000004	+·000005	+·000005	+·000006	+·000005	+·000004	−·000006	−·000005
+·000003	+·000001	+·000006	+·000003	+·000005	+·000002	·000000	+·000001	+·000003	+·000001	−·000004	+·000001	·000000	−·000003
−·000002	−·000004	−·000002	−·000004	+·000001	+·000001	·000000	−·000002	+·000001	+·000004	+·000001	+·000005	+·000001	+·000002

The mean variations $f_{c.1}(h)$, $f_{s.1}(h)$ (lines g and h of Tables 11 to 18), are curved in order in figs. 22 to 37.

15. Extending now the process of elimination described in paragraph 11, so as to further clear the observed variations of their elements $f_{c.1}(h)$, $f_{s.1}(h)$, &c., as shown on lines a and b of Tables 11 to 18, we subtract $f_{c.1}(h)$ from the observed lunar diurnal variation at new moon, commencing with 0 hours; subtract $f_{s.1}(h)$ from the observed variation at first quarter, commencing with 6 hours; add $f_{c.1}(h)$ to the observed variation at full moon, commencing with 12 hours; and add $f_{s.1}(h)$ to the observed variation at last quarter, commencing with 18 hours; thus reducing the residuals formerly obtained to four new residuals, the mean of which may be taken as the residual lunar diurnal variation for the quarters of the moon. A similar process, using lines c and d of Tables 11 to 18, in lieu of lines a and b, leads to the residual lunar diurnal variation for the eighths phases. These residuals, for each quarter of the year and for both the magnetic elements, were found to be practically insignificant, the hourly numbers in all of them lying between the limits $\pm\cdot 000002$. From this we conclude that, in each quarter of the year and for each magnetic element separately, our extended formula practically embodies the whole of the observed lunar diurnal variation for the full lunation.

16. Coefficients of the harmonic equivalents of the typical variations, expressed in the form

$$a_1 \cos \frac{2\pi}{24}h + b_1 \sin \frac{2\pi}{24}h + a_2 \cos 2\left(\frac{2\pi}{24}h\right) + b_2 \sin 2\left(\frac{2\pi}{24}h\right) + \&c.,$$

were calculated for each case, the data being taken from Tables 9 and 10, and from lines g and h of Tables 11 to 18, and their values are exhibited in Tables 19 and 20.

TABLE 19.

Declination.		a_1	b_1	a_2	b_2	A_1	B_1	A_2	B_2
November to January	$f_{c.1}(h)$	+·0000005	−·0000034	+·0000006	+·0000004				
	$f_{s.1}(h)$	...	...	...	...	+·0000023	+·0000023	+·0000010	−·0000003
	$f_{c.2}(h)$	+·0000031	+·0000026	+·0000097	+·0000040				
	$f_{s.2}(h)$	...	...	...	...	−·0000030	+·0000031	−·0000055	+·0000077
February to April	$f_{c.1}(h)$	−·0000032	−·0000010	+·0000009	+·0000009				
	$f_{s.1}(h)$	...	...	...	...	+·0000008	−·0000006	−·0000010	·0000000
	$f_{c.2}(h)$	+·0000014	−·0000022	+·0000023	−·0000009				
	$f_{s.2}(h)$	...	...	...	...	−·0000014	−·0000001	−·0000002	+·0000016
May to July	$f_{c.1}(h)$	−·0000014	−·0000019	−·0000001	−·0000011				
	$f_{s.1}(h)$	...	...	...	...	+·0000029	+·0000008	+·0000002	−·0000010
	$f_{c.2}(h)$	−·0000043	·0000000	−·0000078	+·0000013				
	$f_{s.2}(h)$	...	...	...	...	−·0000024	−·0000049	−·0000017	−·0000080
August to October	$f_{c.1}(h)$	−·0000004	+·0000020	−·0000007	+·0000003				
	$f_{s.1}(h)$	...	...	...	...	−·0000001	−·0000004	−·0000017	−·0000016
	$f_{c.2}(h)$	−·0000054	+·0000008	−·0000076	+·0000047				
	$f_{s.2}(h)$	...	...	...	...	−·0000016	−·0000040	−·0000050	−·0000048

TABLE 20.

Horizontal Force.		a_1	b_1	a_2	b_2	A_1	B_1	A_2	B_2
November to January	$f_{c\cdot 1}(h)$	−·0000020	+·0000015	−·0000021	−·0000013				
	$f_{s\cdot 1}(h)$	...	...	...	...	−·0000050	−·0000023	−·0000023	−·0000012
	$f_{c\cdot 2}(h)$	+·0000006	−·0000093	+·0000001	−·0000126				
	$f_{s\cdot 2}(h)$	...	...	...	...	+·0000136	−·0000017	+·0000134	+·0000007
February to April	$f_{c\cdot 1}(h)$	−·0000038	−·0000004	+·0000018	−·0000006				
	$f_{s\cdot 1}(h)$	...	...	...	...	+·0000015	+·0000003	−·0000014	+·0000025
	$f_{c\cdot 2}(h)$	+·0000014	−·0000036	+·0000011	−·0000067				
	$f_{s\cdot 2}(h)$	...	...	...	...	+·0000123	+·0000018	+·0000071	+·0000010
May to July	$f_{c\cdot 1}(h)$	−·0000023	−·0000003	−·0000010	+·0000025				
	$f_{s\cdot 1}(h)$	...	...	...	...	+·0000028	+·0000026	+·0000013	+·0000028
	$f_{c\cdot 2}(h)$	−·0000021	−·0000032	−·0000014	−·0000021				
	$f_{s\cdot 2}(h)$	...	...	...	...	+·0000053	+·0000017	+·0000044	+·0000007
August to October	$f_{c\cdot 1}(h)$	−·0000028	−·0000017	+·0000013	−·0000006				
	$f_{s\cdot 1}(h)$	...	...	...	...	+·0000033	−·0000017	+·0000008	+·0000008
	$f_{c\cdot 2}(h)$	−·0000004	−·0000037	−·0000003	−·0000033				
	$f_{s\cdot 2}(h)$	...	...	...	...	+·0000067	−·0000013	+·0000063	+·0000010

And from these coefficients were calculated the values of $\frac{1}{2}\sqrt{(a_1+B_1)^2+(b_1-A_1)^2}$; $\frac{1}{2}\sqrt{(a_2+B_2)^2+(b_2-A_2)^2}$; $\frac{1}{2}\sqrt{(a_2-B_2)^2+(b_2+A_2)^2}$; and $\frac{1}{2}\sqrt{(a_1-B_1)^2+(b_1+A_1)^2}$ for each pair of typical variations $f_c(h), f_s(h)$. These quantities are the coefficients A, B, C, D, respectively, of our transformed formula in paragraph 13; and their values are entered in Tables 21 and 22.

TABLE 21.

Declination.	A_1	B_1	C_1	D_1	A_2	B_2	C_2	D_2
November to January	·0000031	·0000003	·0000008	·0000010	·0000042	·0000099	·0000012	·0000002
February to April	·0000021	·0000010	·0000004	·0000013	·0000007	·0000019	·0000006	·0000019
May to July	·0000024	·0000008	·0000006	·0000012	·0000048	·0000080	·0000002	·0000012
August to October	·0000011	·0000015	·0000008	·0000009	·0000049	·0000078	·0000014	·0000008

TABLE 22.

Horizontal force.	A_1	B_1	C_1	D_1	A_2	B_2	C_2	D_2
November to January	·0000038	·0000017	·0000018	·0000017	·0000114	·0000130	·0000054	·0000024
February to April	·0000019	·0000021	·0000010	·0000021	·0000081	·0000070	·0000002	·0000043
May to July	·0000015	·0000011	·0000027	·0000027	·0000042	·0000032	·0000015	·0000021
August to October	·0000033	·0000012	·0000002	·0000009	·0000053	·0000048	·0000016	·0000016

17. Regarding these numbers as the amplitudes of the eight simple waves whose periods are specified in paragraph 13, we see that generally B_2, and in a less degree A_1—which refer to the lunar half-day and the lunar day respectively—form important elements of the whole luni-solar variation; and scarcely less important are the elements that have the A_2's for their amplitudes: the numbers for the remaining five periods are generally smaller, though for none of them are they so small as to approach general insignificance.

18. Up to this point we have, in comparing our formula with observation, had to adapt to the purpose variations obtained for the period of the lunar day. It would obviously be a more direct procedure to arrange the so-called lunar differences of a given season in solar days, according to the age of the moon, and, although the labour of copying out the numbers so arranged, and of re-computing, is considerable, we have thought the verification of the results obtained by the indirect mode of procedure of sufficient importance to justify the undertaking of the task, at least in respect of the element of declination, and for that season, November to January, when the luni-solar variations at Bombay have the largest range. Accordingly, abstract sheets were prepared for the 1st, 2nd, 3rd, &c., solar days after new moon, embracing all such days in the months November to January of the years 1846 to 1870. The hourly sums and means and the diurnal variations were then taken on each abstract sheet, with the results shown in Table 23.

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TABLE 23.—Luni-solar Variations of Declination for each Solar Day after New Moon in the Quarter November to January, expressed in C.G.S. Units of Force.

Bombay astronomical hours.	Days after new moon.														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
h. m.															
12 12	+ ·0000045	+ ·0000028	+ ·0000091	− ·0000072	− ·0000036	− ·0000007	− ·0000054	− ·0000019	+ ·0000013	+ ·0000029	+ ·0000010	+ ·0000011	+ ·0000081	+ ·0000113	+ ·0000086
13 12	+ ·0000010	+ ·0000028	+ ·0000082	− ·0000002	− ·0000023	− ·0000071	− ·0000042	+ ·0000060	− ·0000019	− ·0000125	+ ·0000013	− ·0000009	− ·0000029	+ ·0000076	− ·0000005
14 12	− ·0000007	+ ·0000064	+ ·0000079	+ ·0000112	− ·0000004	− ·0000080	− ·0000027	− ·0000054	+ ·0000027	− ·0000094	− ·0000086	·0000000	− ·0000124	+ ·0000041	·0000000
15 12	+ ·0000017	− ·0000012	+ ·0000093	+ ·0000130	+ ·0000083	− ·0000012	− ·0000028	− ·0000085	− ·0000018	− ·0000108	− ·0000056	− ·0000018	− ·0000101	− ·0000006	− ·0000077
16 12	+ ·0000051	+ ·0000010	+ ·0000065	+ ·0000144	+ ·0000027	+ ·0000038	+ ·0000013	− ·0000075	− ·0000035	− ·0000096	− ·0000059	− ·0000037	− ·0000142	− ·0000043	− ·0000161
17 12	− ·0000068	− ·0000092	+ ·0000067	+ ·0000064	+ ·0000041	+ ·0000015	+ ·0000070	+ ·0000025	+ ·0000028	+ ·0000028	− ·0000060	− ·0000044	− ·0000159	− ·0000069	− ·0000219
18 12	− ·0000101	− ·0000126	− ·0000027	+ ·0000056	+ ·0000097	+ ·0000069	+ ·0000039	+ ·0000049	+ ·0000071	+ ·0000070	− ·0000014	− ·0000033	− ·0000157	− ·0000163	− ·0000229
19 12	− ·0000172	− ·0000219	+ ·0000041	+ ·0000067	− ·0000002	+ ·0000150	+ ·0000191	+ ·0000040	+ ·0000077	+ ·0000148	− ·0000020	− ·0000005	− ·0000130	− ·0000158	− ·0000295
20 12	− ·0000029	− ·0000101	− ·0000017	− ·0000087	+ ·0000059	+ ·0000110	+ ·0000128	+ ·0000066	− ·0000029	+ ·0000118	− ·0000010	+ ·0000062	− ·0000041	− ·0000252	− ·0000244
21 12	− ·0000002	− ·0000020	− ·0000145	− ·0000100	+ ·0000007	+ ·0000060	+ ·0000035	− ·0000052	− ·0000044	+ ·0000023	+ ·0000003	+ ·0000068	+ ·0000063	− ·0000170	− ·0000206
22 12	+ ·0000117	− ·0000082	− ·0000126	− ·0000162	− ·0000135	− ·0000015	− ·0000040	− ·0000125	+ ·0000016	− ·0000031	+ ·0000074	+ ·0000167	+ ·0000234	− ·0000176	+ ·0000028
23 12	+ ·0000091	− ·0000019	− ·0000263	− ·0000050	− ·0000157	− ·0000148	− ·0000093	− ·0000060	− ·0000032	− ·0000004	+ ·0000075	+ ·0000069	+ ·0000211	− ·0000031	+ ·0000171
0 12	+ ·0000163	− ·0000046	− ·0000071	− ·0000156	− ·0000030	− ·0000079	− ·0000139	− ·0000033	− ·0000059	− ·0000005	+ ·0000114	− ·0000016	+ ·0000218	+ ·0000076	+ ·0000183
1 12	+ ·0000283	+ ·0000191	− ·0000016	+ ·0000078	+ ·0000029	− ·0000109	− ·0000164	− ·0000148	− ·0000104	− ·0000132	+ ·0000041	+ ·0000014	+ ·0000209	+ ·0000160	+ ·0000221
2 12	+ ·0000177	+ ·0000285	+ ·0000062	+ ·0000064	− ·0000051	− ·0000127	− ·0000025	− ·0000194	− ·0000113	− ·0000202	+ ·0000014	+ ·0000070	+ ·0000026	+ ·0000187	+ ·0000276
3 12	+ ·0000140	+ ·0000290	+ ·0000147	+ ·0000043	+ ·0000118	+ ·0000053	+ ·0000038	− ·0000071	− ·0000179	− ·0000248	− ·0000110	− ·0000131	− ·0000059	+ ·0000071	+ ·0000113
4 12	− ·0000163	+ ·0000079	+ ·0000165	+ ·0000133	+ ·0000148	+ ·0000238	+ ·0000093	+ ·0000117	− ·0000094	+ ·0000014	− ·0000164	− ·0000135	− ·0000239	− ·0000012	− ·0000151
5 12	− ·0000165	− ·0000129	− ·0000036	+ ·0000099	+ ·0000082	+ ·0000125	+ ·0000134	+ ·0000112	+ ·0000087	+ ·0000131	+ ·0000019	− ·0000111	− ·0000214	− ·0000130	− ·0000059
6 12	− ·0000135	− ·0000070	− ·0000112	+ ·0000033	− ·0000024	+ ·0000032	− ·0000001	+ ·0000161	+ ·0000114	+ ·0000138	+ ·0000016	− ·0000035	+ ·0000004	+ ·0000093	+ ·0000068
7 12	− ·0000100	− ·0000071	− ·0000043	− ·0000027	− ·0000064	− ·0000011	+ ·0000019	+ ·0000112	+ ·0000065	+ ·0000148	+ ·0000031	− ·0000022	+ ·0000031	+ ·0000045	+ ·0000157
8 12	− ·0000129	− ·0000059	− ·0000046	− ·0000126	− ·0000022	− ·0000041	− ·0000064	+ ·0000059	+ ·0000107	+ ·0000088	+ ·0000106	+ ·0000019	+ ·0000135	+ ·0000141	+ ·0000061
9 12	− ·0000031	+ ·0000027	− ·0000019	− ·0000108	− ·0000043	− ·0000063	− ·0000050	+ ·0000023	+ ·0000027	+ ·0000102	+ ·0000010	+ ·0000080	+ ·0000035	+ ·0000087	+ ·0000062
10 12	− ·0000032	+ ·0000019	+ ·0000015	− ·0000069	− ·0000062	− ·0000054	− ·0000005	+ ·0000083	+ ·0000046	+ ·0000050	− ·0000015	+ ·0000047	+ ·0000093	+ ·0000067	+ ·0000100
11 12	+ ·0000049	+ ·0000033	+ ·0000022	− ·0000063	− ·0000033	− ·0000062	− ·0000019	+ ·0000005	+ ·0000047	− ·0000047	+ ·0000066	− ·0000003	+ ·0000063	+ ·0000055	+ ·0000113

TABLE 23—*continued*.

Bombay astronomical hours.		Days after new moon.														
h.	m.	16	17	18	19	20	21	−9	−8	−7	−6	−5	−4	−3	−2	−1
12	12	+ ·0000077	− ·0000029	− ·0000001	− ·0000017	− ·0000014	− ·0000056	+ ·0000018	− ·0000060	− ·0000064	+ ·0000014	− ·0000097	+ ·0000070	− ·0000010	+ ·0000016	+ ·0000060
13	12	+ ·0000108	− ·0000052	+ ·0000006	− ·0000021	− ·0000048	+ ·0000051	− ·0000004	− ·0000023	− ·0000095	− ·0000013	− ·0000060	+ ·0000050	− ·0000039	+ ·0000060	+ ·0000111
14	12	+ ·0000072	− ·0000033	+ ·0000052	− ·0000025	− ·0000005	+ ·0000062	+ ·0000013	− ·0000016	− ·0000077	+ ·0000072	− ·0000034	+ ·0000026	− ·0000063	− ·0000012	+ ·0000101
15	12	− ·0000015	− ·0000068	+ ·0000098	+ ·0000071	+ ·0000108	+ ·0000029	+ ·0000086	+ ·0000061	− ·0000056	+ ·0000037	− ·0000076	+ ·0000005	+ ·0000023	− ·0000009	+ ·0000031
16	12	− ·0000100	− ·0000063	+ ·0000118	+ ·0000081	+ ·0000059	+ ·0000067	+ ·0000068	+ ·0000002	+ ·0000010	+ ·0000019	− ·0000013	+ ·0000044	− ·0000074	·0000000	+ ·0000037
17	12	− ·0000061	− ·0000064	+ ·0000055	+ ·0000116	+ ·0000091	+ ·0000006	+ ·0000042	+ ·0000045	+ ·0000037	+ ·0000086	− ·0000005	− ·0000034	− ·0000051	− ·0000035	+ ·0000022
18	12	− ·0000161	− ·0000116	+ ·0000006	+ ·0000053	+ ·0000112	+ ·0000019	+ ·0000021	+ ·0000103	+ ·0000067	+ ·0000123	+ ·0000109	− ·0000015	+ ·0000006	+ ·0000013	− ·0000126
19	12	− ·0000171	− ·0000133	− ·0000146	+ ·0000038	+ ·0000103	+ ·0000177	+ ·0000013	+ ·0000194	+ ·0000082	+ ·0000163	+ ·0000048	+ ·0000035	− ·0000033	− ·0000063	− ·0000289
20	12	− ·0000053	− ·0000085	− ·0000191	− ·0000010	− ·0000083	+ ·0000193	+ ·0000027	+ ·0000080	+ ·0000132	+ ·0000187	+ ·0000055	+ ·0000020	+ ·0000035	− ·0000051	− ·0000196
21	12	− ·0000141	− ·0000133	− ·0000204	− ·0000069	+ ·0000004	+ ·0000095	+ ·0000014	− ·0000019	+ ·0000214	+ ·0000157	+ ·0000155	+ ·0000243	+ ·0000164	− ·0000044	+ ·0000036
22	12	− ·0000003	− ·0000067	− ·0000194	− ·0000164	− ·0000143	− ·0000068	− ·0000004	− ·0000027	+ ·0000202	+ ·0000147	+ ·0000234	+ ·0000151	+ ·0000236	+ ·0000007	− ·0000081
23	12	+ ·0000020	+ ·0000048	− ·0000167	− ·0000192	− ·0000009	− ·0000141	+ ·0000002	− ·0000068	+ ·0000082	− ·0000125	+ ·0000180	+ ·0000081	+ ·0000191	+ ·0000109	+ ·0000117
0	12	+ ·0000004	+ ·0000028	− ·0000108	− ·0000112	− ·0000132	− ·0000215	− ·0000043	− ·0000096	− ·0000061	− ·0000171	+ ·0000070	+ ·0000033	+ ·0000258	+ ·0000176	+ ·0000181
1	12	+ ·0000043	+ ·0000262	− ·0000085	− ·0000106	− ·0000056	− ·0000218	− ·0000291	+ ·0000010	− ·0000116	− ·0000253	+ ·0000014	− ·0000053	+ ·0000247	+ ·0000117	+ ·0000274
2	12	+ ·0000158	+ ·0000387	+ ·0000158	+ ·0000083	+ ·0000017	− ·0000192	− 0000176	− ·0000216	− ·0000135	− ·0000230	− ·0000128	− ·0000085	+ ·0000023	− ·0000072	+ ·0000211
3	12	+ ·0000146	+ ·0000247	+ ·0000232	+ ·0000198	+ ·0000020	− ·0000086	− 0000108	− ·0000159	− ·0000138	− ·0000132	− ·0000141	− ·0000273	− ·0000074	− ·0000022	+ ·0000087
4	12	+ ·0000053	+ ·0000075	+ ·0000212	+ ·0000130	+ ·0000142	+ ·0000211	+ ·0000056	+ ·0000129	− ·0000108	− ·0000211	− ·0000063	− ·0000237	− ·0000238	− ·0000104	− ·0000092
5	12	− ·0000130	− ·0000062	+ ·0000094	+ ·0000091	+ ·0000160	+ ·0000190	+ ·0000100	+ ·0000146	+ ·0000022	− ·0000007	− ·0000074	− ·0000098	− ·0000169	− ·0000060	− ·0000060
6	12	− ·0000039	+ ·0000001	+ ·0000021	− ·0000011	− ·0000044	− ·0000013	+ ·0000044	+ ·0000017	− ·0000018	− ·0000019	+ ·0000031	− ·0000047	− ·0000126	+ ·0000005	− ·0000154
7	12	− ·0000009	− ·0000046	+ ·0000065	− ·0000035	− ·0000091	+ ·0000005	+ ·0000005	+ ·0000014	− ·0000007	+ ·0000042	− ·0000058	− ·0000007	− ·0000068	+ ·0000017	− ·0000118
8	12	+ ·0000038	− ·0000062	− ·0000017	− ·0000081	− ·0000090	− ·0000043	+ ·0000027	+ ·0000028	+ ·0000007	+ ·0000011	− ·0000068	+ ·0000056	− ·0000075	− ·0000013	− ·0000069
9	12	+ ·0000043	− ·0000026	− ·0000043	− ·0000029	− ·0000022	+ ·0000018	+ ·0000030	− ·0000021	+ ·0000055	+ ·0000055	+ ·0000012	− ·0000027	− ·0000070	− ·0000001	+ ·0000011
10	12	+ ·0000071	·0000000	+ ·0000036	− ·0000067	− ·0000090	− ·0000028	+ ·0000022	− ·0000037	− ·0000006	+ ·0000001	− ·0000082	− ·0000011	− ·0000070	− ·0000011	− ·0000087
11	12	+ ·0000045	− ·0000016	+ ·0000001	− ·0000063	+ ·0000017	− ·0000066	+ ·0000045	− ·0000085	− ·0000030	+ ·0000038	− ·0000017	+ ·0000067	− ·0000030	− ·0000028	+ ·0000001

These variations are curved in thin lines in fig. 48, being numbered 1 to 20 and −9 to −1 in order.

19. In calculating luni-solar variations for each day of the moon's age by our extended formula, we carried the harmonic equivalents of the typical variations of lines 1 and 5 of Tables 9 and 10, and of lines g and h of Tables 11 and 15, to six pairs of terms, and used these equivalents instead of the typical variations from which they were derived. The coefficients found were, for November to January—

TABLE 24.—Declination.

	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	a_5	b_5	a_6	b_6
$f_{c \cdot 1}(h)$	+ ·0000005	− ·0000034	+ ·0000006	+ ·0000004	− ·0000002	− ·0000009	− ·0000002	− ·0000006	+ ·0000012	+ ·0000016	+ ·0000011	+ ·0000006
$f_{s \cdot 1}(h)$	+ ·0000023	+ ·0000023	+ ·0000010	− ·0000003	− ·0000004	+ ·0000001	+ ·0000008	·0000000	− ·0000004	− ·0000005	− ·0000002	+ ·0000001
$f_{c \cdot 2}(h)$	+ ·0000031	+ ·0000026	+ ·0000097	+ ·0000040	+ ·0000033	− ·0000031	+ ·0000001	+ ·0000024	− ·0000007	+ ·0000003	·0000000	− ·0000008
$f_{s \cdot 2}(h)$	− ·0000030	+ ·0000031	− ·0000055	+ ·0000077	− ·0000023	+ ·0000023	+ ·0000001	− ·0000009	+ ·0000008	− ·0000015	+ ·0000005	+ ·0000001

TABLE 25.—Horizontal Force.

	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	a_5	b_5	a_6	b_6
$f_{c \cdot 1}(h)$	− ·0000020	+ ·0000015	− ·0000021	− ·0000013	+ ·0000005	+ ·0000007	·0000000	+ ·0000013	+ ·0000003	− ·0000002	− ·0000019	− ·0000001
$f_{s \cdot 1}(h)$	− ·0000050	− ·0000023	− ·0000023	− ·0000012	+ ·0000004	+ ·0000002	+ ·0000006	− ·0000011	− ·0000008	− ·0000005	− ·0000001	− ·0000008
$f_{c \cdot 2}(h)$	+ ·0000006	− ·0000093	+ ·0000001	− ·0000126	− ·0000009	− ·0000053	− ·0000009	− ·0000019	− ·0000006	+ ·0000003	− ·0000004	+ ·0000015
$f_{s \cdot 2}(h)$	+ ·0000136	− ·0000017	+ ·0000134	+ ·0000007	+ ·0000038	− ·0000028	+ ·0000005	− ·0000009	− ·0000009	− ·0000011	+ ·0000009	− ·0000010

20. And now, using 12 as the hour of occurrence of new moon on the initial solar day, the luni-solar variations found were as follows :—

TABLE 26.—Declination. November to January. Values, for each Solar Hour of the Moon's Age, of

$$f_{c.1}(h)\cos\left(2\pi\frac{t}{P}\right)+f_{s.1}(h)\sin\left(2\pi\frac{t}{P}\right)+f_{c.2}(h)\cos 2\left(2\pi\frac{t}{P}\right)+f_{s.2}(h)\sin 2\left(2\pi\frac{t}{P}\right).$$

Bombay astronomical hours.	Days after new moon.														
	−9	−8	−7	−6	−5	−4	−3	−2	−1	+1	+2	+3	+4	+5	+6
12	−·0000036	−·0000042	−·0000040	−·0000033	−·0000020	−·0000002	+·0000015	+·0000030	+·0000039	+·0000041	+·0000035	+·0000024	+·0000009	−·0000012	−·0000024
13	−·0000011	−·0000022	−·0000029	−·0000030	−·0000026	−·0000015	·0000000	+·0000014	+·0000026	+·0000034	+·0000037	+·0000031	+·0000020	+·0000006	−·0000012
14	+·0000023	+·0000012	+·0000003	−·0000002	−·0000002	+·0000003	+·0000012	+·0000024	+·0000034	+·0000041	+·0000044	+·0000039	+·0000027	+·0000010	−·0000013
15	+·0000056	+·0000053	+·0000047	+·0000040	+·0000035	+·0000029	+·0000038	+·0000040	+·0000044	+·0000050	+·0000051	+·0000048	+·0000038	+·0000021	−·0000002
16	+·0000093	+·0000089	+·0000077	+·0000056	+·0000036	+·0000018	+·0000007	+·0000003	+·0000008	+·0000019	+·0000033	+·0000051	+·0000054	+·0000052	+·0000041
17	+·0000125	+·0000115	+·0000088	+·0000048	+·0000007	−·0000033	−·0000062	−·0000075	−·0000073	−·0000054	−·0000023	+·0000012	+·0000044	+·0000070	+·0000080
18	+·0000129	+·0000120	+·0000092	+·0000048	−·0000002	−·0000051	−·0000088	−·0000111	−·0000114	−·0000099	−·0000066	−·0000023	+·0000020	+·0000049	+·0000085
19	+·0000103	+·0000120	+·0000118	+·0000096	+·0000064	+·0000021	−·0000021	−·0000057	−·0000077	−·0000082	−·0000070	−·0000043	−·0000009	+·0000027	+·0000059
20	+·0000055	+·0000109	+·0000146	+·0000161	+·0000152	+·0000122	+·0000076	+·0000026	−·0000023	−·0000059	−·0000078	−·0000078	−·0000046	−·0000042	+·0000018
21	−·0000006	+·0000070	+·0000132	+·0000172	+·0000183	+·0000167	+·0000125	+·0000063	−·0000003	−·0000061	−·0000076	−·0000124	−·0000117	−·0000084	−·0000035
22	−·0000079	−·0000013	+·0000053	+·0000107	+·0000139	+·0000146	+·0000125	+·0000082	+·0000024	−·0000038	−·0000091	−·0000128	−·0000138	−·0000123	−·0000082
23	−·0000155	−·0000108	−·0000048	+·0000020	+·0000081	+·0000122	+·0000142	+·0000135	+·0000103	+·0000055	+·0000004	−·0000058	−·0000099	−·0000118	−·0000114
0	−·0000217	−·0000192	−·0000139	−·0000057	+·0000029	+·0000107	+·0000167	+·0000199	+·0000199	+·0000165	+·0000107	+·0000035	−·0000039	−·0000099	−·0000135
1	−·0000237	−·0000245	−·0000195	−·0000123	−·0000035	+·0000057	+·0000139	+·0000195	+·0000217	+·0000202	+·0000153	+·0000079	−·0000005	−·0000082	−·0000141
2	−·0000179	−·0000211	−·0000216	−·0000184	−·0000125	−·0000051	+·0000028	+·0000095	+·0000139	+·0000154	+·0000139	+·0000094	+·0000032	−·0000035	−·0000095
3	−·0000056	−·0000120	−·0000168	−·0000189	−·0000184	−·0000153	−·0000102	−·0000039	+·0000024	+·0000075	+·0000111	+·0000120	+·0000105	+·0000071	+·0000025
4	+·0000074	+·0000009	−·0000063	−·0000127	−·0000172	−·0000189	−·0000177	−·0000137	−·0000085	−·0000002	+·0000069	+·0000128	+·0000164	+·0000169	+·0000147
5	+·0000128	+·0000087	+·0000026	−·0000043	−·0000106	−·0000152	−·0000173	−·0000165	−·0000128	−·0000069	+·0000002	+·0000075	+·0000135	+·0000173	+·0000181
6	+·0000096	+·0000078	+·0000044	−·0000002	−·0000050	−·0000095	−·0000125	−·0000137	−·0000132	−·0000104	−·0000062	−·0000011	+·0000039	+·0000082	+·0000109
7	+·0000044	+·0000037	+·0000021	−·0000001	−·0000027	−·0000055	−·0000080	−·0000097	−·0000105	−·0000101	−·0000087	−·0000063	−·0000035	−·0000002	+·0000025
8	+·0000021	+·0000017	+·0000009	−·0000003	−·0000016	−·0000033	−·0000049	−·0000058	−·0000072	−·0000075	−·0000073	−·0000063	−·0000048	−·0000027	−·0000006
9	+·0000019	+·0000020	+·0000018	+·0000011	+·0000002	−·0000012	−·0000025	−·0000037	−·0000048	−·0000052	−·0000056	−·0000052	−·0000043	−·0000029	−·0000014
10	−·0000002	+·0000006	+·0000013	+·0000018	+·0000020	+·0000018	+·0000012	+·0000001	−·0000012	−·0000024	−·0000036	−·0000043	−·0000043	−·0000039	−·0000031
11	−·0000034	−·0000025	−·0000012	+·0000005	+·0000019	+·0000031	+·0000037	+·0000035	+·0000026	+·0000012	−·0000003	−·0000022	−·0000036	−·0000044	−·0000047

TABLE 26—continued.

Bombay astronomical hours.	Days after new moon.													
	+ 7	+ 8	+ 9	+ 10	+ 11	+ 12	+ 13	+ 14	+ 15	+ 16	+ 17	+ 18	+ 19	+ 20
12	− ·0000035	− ·0000040	− ·0000038	− ·0000028	− ·0000013	+ ·0000004	+ ·0000021	+ ·0000033	+ ·0000040	+ ·0000041	+ ·0000034	+ ·0000021	+ ·0000003	− ·0000015
13	− ·0000027	− ·0000039	− ·0000044	− ·0000043	− ·0000034	− ·0000022	− ·0000005	+ ·0000012	+ ·0000026	+ ·0000036	+ ·0000039	+ ·0000035	+ ·0000026	+ ·0000013
14	− ·0000035	− ·0000054	− ·0000068	− ·0000074	− ·0000074	− ·0000063	− ·0000046	− ·0000025	− ·0000003	+ ·0000017	+ ·0000032	+ ·0000041	+ ·0000043	+ ·0000038
15	− ·0000026	− ·0000054	− ·0000077	− ·0000096	− ·0000106	− ·0000107	− ·0000097	− ·0000080	− ·0000057	− ·0000029	− ·0000002	+ ·0000023	+ ·0000041	+ ·0000052
16	+ ·0000020	− ·0000011	− ·0000045	− ·0000079	− ·0000109	− ·0000127	− ·0000134	− ·0000125	− ·0000103	− ·0000069	− ·0000029	+ ·0000012	+ ·0000048	+ ·0000073
17	+ ·0000074	+ ·0000051	+ ·0000013	− ·0000015	− ·0000071	− ·0000107	− ·0000125	− ·0000127	− ·0000109	− ·0000074	− ·0000027	+ ·0000022	+ ·0000069	+ ·0000104
18	+ ·0000092	+ ·0000080	+ ·0000052	+ ·0000013	− ·0000031	− ·0000070	− ·0000096	− ·0000107	− ·0000097	− ·0000068	− ·0000027	+ ·0000021	+ ·0000068	+ ·0000106
19	+ ·0000078	+ ·0000081	+ ·0000068	+ ·0000039	·0000000	− ·0000043	− ·0000082	− ·0000109	− ·0000120	− ·0000112	− ·0000088	− ·0000047	+ ·0000001	+ ·0000048
20	+ ·0000055	+ ·0000084	+ ·0000093	+ ·0000083	+ ·0000053	+ ·0000006	− ·0000047	− ·0000101	− ·0000144	− ·0000169	− ·0000170	− ·0000147	− ·0000102	− ·0000043
21	+ ·0000025	+ ·0000080	+ ·0000120	+ ·0000138	+ ·0000128	+ ·0000092	+ ·0000036	− ·0000033	− ·0000101	− ·0000157	− ·0000190	− ·0000194	− ·0000168	− ·0000116
22	− ·0000024	+ ·0000042	+ ·0000101	+ ·0000145	+ ·0000166	+ ·0000157	+ ·0000123	+ ·0000065	− ·0000003	− ·0000072	− ·0000129	− ·0000165	− ·0000173	− ·0000154
23	− ·0000086	− ·0000036	+ ·0000021	+ ·0000080	+ ·0000128	+ ·0000155	+ ·0000155	+ ·0000129	+ ·0000080	+ ·0000016	− ·0000053	− ·0000114	− ·0000159	− ·0000177
0	− ·0000152	− ·0000120	− ·0000072	− ·0000007	+ ·0000059	+ ·0000117	+ ·0000153	+ ·0000160	+ ·0000137	+ ·0000083	+ ·0000011	− ·0000066	− ·0000140	− ·0000194
1	− ·0000170	− ·0000164	− ·0000123	− ·0000058	+ ·0000021	+ ·0000099	+ ·0000161	+ ·0000193	+ ·0000191	+ ·0000154	+ ·0000086	+ ·0000001	− ·0000091	− ·0000169
2	− ·0000136	− ·0000149	− ·0000130	− ·0000084	− ·0000015	+ ·0000062	+ ·0000135	+ ·0000187	+ ·0000213	+ ·0000207	+ ·0000167	+ ·0000098	+ ·0000014	− ·0000073
3	− ·0000024	− ·0000068	− ·0000093	− ·0000097	− ·0000076	− ·0000036	+ ·0000020	+ ·0000076	+ ·0000127	+ ·0000160	+ ·0000172	+ ·0000155	+ ·0000114	+ ·0000052
4	+ ·0000102	+ ·0000040	− ·0000027	− ·0000083	− ·0000123	− ·0000136	− ·0000122	− ·0000082	− ·0000026	+ ·0000040	+ ·0000097	+ ·0000138	+ ·0000155	+ ·0000143
5	+ ·0000160	+ ·0000113	+ ·0000049	− ·0000021	− ·0000087	− ·0000133	− ·0000154	− ·0000147	− ·0000112	− ·0000057	+ ·0000008	+ ·0000071	+ ·0000118	+ ·0000144
6	+ ·0000119	+ ·0000109	+ ·0000082	+ ·0000045	+ ·0000001	− ·0000038	− ·0000065	− ·0000076	− ·0000070	− ·0000049	− ·0000013	+ ·0000026	+ ·0000062	+ ·0000088
7	+ ·0000049	+ ·0000064	+ ·0000069	+ ·0000067	+ ·0000055	+ ·0000041	+ ·0000026	+ ·0000014	+ ·0000007	+ ·0000007	+ ·0000011	+ ·0000020	+ ·0000031	+ ·0000040
8	+ ·0000014	+ ·0000032	+ ·0000045	+ ·0000052	+ ·0000054	+ ·0000051	+ ·0000046	+ ·0000039	+ ·0000031	+ ·0000024	+ ·0000021	+ ·0000020	+ ·0000020	+ ·0000020
9	+ ·0000002	+ ·0000018	+ ·0000028	+ ·0000035	+ ·0000037	+ ·0000036	+ ·0000031	+ ·0000024	+ ·0000018	+ ·0000013	+ ·0000011	+ ·0000010	+ ·0000012	+ ·0000014
10	− ·0000018	− ·0000003	+ ·0000011	+ ·0000025	+ ·0000031	+ ·0000036	+ ·0000035	+ ·0000029	+ ·0000019	+ ·0000009	− ·0000001	− ·0000008	− ·0000012	− ·0000010
11	− ·0000040	− ·0000027	− ·0000010	+ ·0000008	+ ·0000024	+ ·0000039	+ ·0000046	+ ·0000044	+ ·0000036	+ ·0000022	+ ·0000004	− ·0000011	− ·0000026	− ·0000034

TABLE 27.—Horizontal Force. November to January. Values, for each Solar Hour of the Moon's Age, of

$$f_{c.1}(h)\cos\left(2\pi\frac{t}{\mathrm{P}}\right)+f_{s.1}(h)\sin\left(2\pi\frac{t}{\mathrm{P}}\right)+f_{c.2}(h)\cos 2\left(2\pi\frac{t}{\mathrm{P}}\right)+f_{s.2}(h)\sin 2\left(2\pi\frac{t}{\mathrm{P}}\right).$$

Bombay astronomical hours.	Days after new moon.														
	−9	−8	−7	−6	−5	−4	−3	−2	−1	+1	+2	+3	+4	+5	+6
12	− ·0000034	− ·0000033	− ·0000032	− ·0000030	− ·0000029	− ·0000028	− ·0000029	− ·0000029	− ·0000032	− ·0000031	− ·0000028	− ·0000025	− ·0000017	− ·0000008	+ ·0000005
13	− ·0000019	− ·0000012	− ·0000007	− ·0000001	+ ·0000002	+ ·0000002	·0000000	− ·0000005	− ·0000009	− ·0000014	− ·0000018	− ·0000018	− ·0000015	− ·0000010	·0000000
14	− ·0000014	+ ·0000005	+ ·0000022	+ ·0000034	+ ·0000037	+ ·0000035	+ ·0000025	+ ·0000011	− ·0000002	− ·0000014	− ·0000021	− ·0000022	− ·0000016	− ·0000005	+ ·0000010
15	− ·0000018	·0000000	+ ·0000014	+ ·0000022	+ ·0000021	+ ·0000012	− ·0000003	− ·0000022	− ·0000039	− ·0000051	− ·0000055	− ·0000050	− ·0000035	− ·0000012	+ ·0000021
16	− ·0000067	− ·0000048	− ·0000027	− ·0000008	+ ·0000003	+ ·0000005	− ·0000001	− ·0000017	− ·0000036	− ·0000053	− ·0000065	− ·0000069	− ·0000059	− ·0000032	− ·0000008
17	− ·0000138	− ·0000100	− ·0000051	− ·0000003	+ ·0000037	+ ·0000063	+ ·0000071	+ ·0000060	+ ·0000037	+ ·0000004	− ·0000030	− ·0000061	− ·0000068	− ·0000062	− ·0000039
18	− ·0000170	− ·0000116	− ·0000048	+ ·0000021	+ ·0000077	+ ·0000118	+ ·0000134	+ ·0000126	+ ·0000098	+ ·0000054	+ ·0000006	− ·0000037	− ·0000066	− ·0000073	− ·0000059
19	− ·0000163	− ·0000125	− ·0000049	− ·0000001	+ ·0000061	+ ·0000113	+ ·0000143	+ ·0000150	+ ·0000130	+ ·0000090	+ ·0000036	− ·0000020	− ·0000068	− ·0000099	− ·0000109
20	− ·0000167	− ·0000179	− ·0000156	− ·0000106	− ·0000038	+ ·0000041	+ ·0000110	+ ·0000157	+ ·0000175	+ ·0000161	+ ·0000136	+ ·0000046	− ·0000033	− ·0000109	− ·0000169
21	− ·0000165	− ·0000236	− ·0000260	− ·0000238	− ·0000171	− ·0000072	+ ·0000040	+ ·0000144	+ ·0000218	+ ·0000255	+ ·0000236	+ ·0000175	+ ·0000077	− ·0000035	− ·0000149
22	− ·0000076	− ·0000201	− ·0000295	− ·0000321	− ·0000295	− ·0000219	− ·0000102	+ ·0000032	+ ·0000158	+ ·0000250	+ ·0000298	+ ·0000287	+ ·0000221	+ ·0000113	− ·0000020
23	+ ·0000091	− ·0000062	− ·0000201	− ·0000300	− ·0000344	− ·0000327	− ·0000253	− ·0000135	+ ·0000001	+ ·0000131	+ ·0000233	+ ·0000287	+ ·0000281	+ ·0000221	+ ·0000112
0	+ ·0000243	+ ·0000110	− ·0000034	− ·0000164	− ·0000259	− ·0000303	− ·0000292	− ·0000229	− ·0000130	− ·0000016	+ ·0000097	+ ·0000182	+ ·0000223	+ ·0000213	+ ·0000154
1	+ ·0000312	+ ·0000233	+ ·0000125	− ·0000005	− ·0000104	− ·0000185	− ·0000226	− ·0000223	− ·0000180	− ·0000106	− ·0000017	+ ·0000070	+ ·0000134	+ ·0000161	+ ·0000149
2	+ ·0000323	+ ·0000287	+ ·0000211	+ ·0000108	− ·0000005	− ·0000107	− ·0000184	− ·0000222	− ·0000217	− ·0000173	− ·0000099	− ·0000009	+ ·0000076	+ ·0000139	+ ·0000171
3	+ ·0000285	+ ·0000276	+ ·0000224	+ ·0000140	+ ·0000037	− ·0000067	− ·0000152	− ·0000220	− ·0000241	− ·0000221	− ·0000164	− ·0000082	+ ·0000009	+ ·0000093	+ ·0000152
4	+ ·0000192	+ ·0000210	+ ·0000197	+ ·0000152	+ ·0000085	+ ·0000003	− ·0000076	− ·0000146	− ·0000177	− ·0000204	− ·0000178	− ·0000142	− ·0000076	− ·0000006	+ ·0000058
5	+ ·0000067	+ ·0000120	+ ·0000156	+ ·0000170	+ ·0000151	+ ·0000146	+ ·0000061	− ·0000005	− ·0000068	− ·0000119	− ·0000148	− ·0000152	− ·0000132	− ·0000088	− ·0000034
6	− ·0000017	+ ·0000051	+ ·0000113	+ ·0000155	+ ·0000172	+ ·0000164	+ ·0000129	+ ·0000077	+ ·0000017	− ·0000040	− ·0000082	− ·0000106	− ·0000105	− ·0000081	− ·0000040
7	− ·0000050	+ ·0000010	+ ·0000064	+ ·0000104	+ ·0000126	+ ·0000127	+ ·0000108	+ ·0000074	+ ·0000032	− ·0000012	− ·0000042	− ·0000059	− ·0000058	− ·0000040	− ·0000008
8	− ·0000065	− ·0000012	+ ·0000040	+ ·0000083	+ ·0000110	+ ·0000120	+ ·0000111	+ ·0000085	+ ·0000048	·0000000	− ·0000026	− ·0000049	− ·0000059	− ·0000050	− ·0000028
9	− ·0000093	− ·0000033	+ ·0000029	+ ·0000087	+ ·0000129	+ ·0000152	+ ·0000150	+ ·0000127	+ ·0000086	+ ·0000036	− ·0000012	− ·0000052	− ·0000076	− ·0000080	− ·0000064
10	− ·0000094	− ·0000040	+ ·0000017	+ ·0000068	+ ·0000105	+ ·0000125	+ ·0000125	+ ·0000105	+ ·0000070	+ ·0000027	− ·0000010	− ·0000042	− ·0000058	− ·0000068	− ·0000040
11	− ·0000065	− ·0000040	− ·0000015	+ ·0000007	+ ·0000022	+ ·0000030	+ ·0000027	+ ·0000019	+ ·0000005	− ·0000010	− ·0000023	− ·0000028	− ·0000025	− ·0000015	+ ·0000005

TABLE 27—continued.

Bombay astro-nomical hours.	Days after new moon.													
	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18	+19	+20
12	+ ·0000020	+ ·0000034	+ ·0000045	− ·0000012	+ ·0000062	+ ·0000062	+ ·0000058	+ ·0000049	+ ·0000036	+ ·0000021	+ ·0000007	− ·0000007	− ·0000019	− ·0000027
13	+ ·0000009	+ ·0000020	+ ·0000029	+ ·0000034	+ ·0000036	+ ·0000034	+ ·0000029	+ ·0000020	+ ·0000007	− ·0000003	− ·0000012	− ·0000020	− ·0000024	− ·0000024
14	+ ·0000026	+ ·0000040	+ ·0000049	+ ·0000050	+ ·0000043	+ ·0000030	+ ·0000015	− ·0000012	− ·0000033	− ·0000050	− ·0000060	− ·0000063	− ·0000057	− ·0000043
15	+ ·0000045	+ ·0000070	+ ·0000085	+ ·0000090	+ ·0000085	+ ·0000067	+ ·0000042	+ ·0000013	− ·0000016	− ·0000041	− ·0000055	− ·0000063	− ·0000060	− ·0000046
16	+ ·0000029	+ ·0000066	+ ·0000099	+ ·0000123	+ ·0000133	+ ·0000128	+ ·0000108	+ ·0000076	+ ·0000039	− ·0000003	− ·0000037	− ·0000066	− ·0000082	− ·0000085
17	·0000000	+ ·0000046	+ ·0000093	+ ·0000133	+ ·0000157	+ ·0000161	+ ·0000143	+ ·0000104	+ ·0000049	− ·0000013	− ·0000072	− ·0000121	− ·0000155	− ·0000164
18	− ·0000025	+ ·0000023	+ ·0000073	+ ·0000118	+ ·0000148	+ ·0000156	+ ·0000137	+ ·0000095	+ ·0000033	− ·0000039	− ·0000107	− ·0000164	− ·0000200	− ·0000189
19	− ·0000093	− ·0000058	− ·0000009	+ ·0000045	+ ·0000094	+ ·0000126	+ ·0000136	+ ·0000122	+ ·0000083	+ ·0000023	− ·0000038	− ·0000098	− ·0000145	− ·0000173
20	− ·0000201	− ·0000197	− ·0000161	− ·0000096	− ·0000015	+ ·0000069	+ ·0000140	+ ·0000189	+ ·0000206	+ ·0000187	+ ·0000139	+ ·0000067	− ·0000014	− ·0000091
21	− ·0000240	− ·0000292	− ·0000294	− ·0000247	− ·0000156	− ·0000037	+ ·0000085	+ ·0000197	+ ·0000275	+ ·0000308	+ ·0000289	+ ·0000222	+ ·0000120	+ ·0000001
22	− ·0000154	− ·0000255	− ·0000326	− ·0000333	− ·0000283	− ·0000181	− ·0000049	+ ·0000097	+ ·0000228	+ ·0000321	+ ·0000358	+ ·0000337	+ ·0000259	+ ·0000138
23	− ·0000022	− ·0000157	− ·0000266	− ·0000333	− ·0000341	− ·0000286	− ·0000180	− ·0000040	+ ·0000115	+ ·0000254	+ ·0000353	+ ·0000396	+ ·0000375	+ ·0000294
0	+ ·0000054	− ·0000066	− ·0000181	− ·0000274	− ·0000324	− ·0000318	− ·0000255	− ·0000148	− ·0000008	+ ·0000139	+ ·0000282	+ ·0000362	+ ·0000400	+ ·0000381
1	+ ·0000097	+ ·0000014	− ·0000085	− ·0000180	− ·0000253	− ·0000291	− ·0000281	− ·0000227	− ·0000130	− ·0000010	+ ·0000118	+ ·0000227	+ ·0000313	+ ·0000351
2	+ ·0000158	+ ·0000112	+ ·0000033	− ·0000062	− ·0000156	− ·0000232	− ·0000272	− ·0000268	− ·0000220	− ·0000133	− ·0000021	+ ·0000097	+ ·0000207	+ ·0000285
3	+ ·0000177	+ ·0000162	+ ·0000113	+ ·0000039	− ·0000051	− ·0000134	− ·0000197	− ·0000225	− ·0000212	− ·0000160	− ·0000077	+ ·0000033	+ ·0000128	+ ·0000215
4	+ ·0000106	+ ·0000130	+ ·0000125	+ ·0000093	+ ·0000041	− ·0000012	− ·0000078	− ·0000120	− ·0000138	− ·0000128	− ·0000089	− ·0000032	+ ·0000041	+ ·0000112
5	+ ·0000022	+ ·0000066	+ ·0000105	+ ·0000116	+ ·0000102	+ ·0000067	+ ·0000017	− ·0000037	− ·0000086	− ·0000120	− ·0000132	− ·0000172	− ·0000100	− ·0000029
6	+ ·0000010	+ ·0000060	+ ·0000099	+ ·0000117	+ ·0000139	+ ·0000083	+ ·0000034	− ·0000063	− ·0000088	− ·0000140	− ·0000174	− ·0000182	− ·0000162	− ·0000118
7	+ ·0000030	+ ·0000070	+ ·0000098	+ ·0000108	+ ·0000102	+ ·0000076	+ ·0000031	− ·0000023	− ·0000087	− ·0000127	− ·0000161	− ·0000172	− ·0000160	− ·0000129
8	+ ·0000005	+ ·0000010	+ ·0000071	+ ·0000091	+ ·0000094	+ ·0000080	+ ·0000048	+ ·0000004	+ ·0000004	− ·0000070	− ·0000092	− ·0000149	− ·0000149	− ·0000129
9	− ·0000032	+ ·0000009	+ ·0000050	+ ·0000082	+ ·0000099	+ ·0000095	+ ·0000071	+ ·0000029	− ·0000023	− ·0000079	− ·0000128	− ·0000160	− ·0000170	− ·0000158
10	− ·0000010	+ ·0000028	+ ·0000063	+ ·0000089	+ ·0000101	+ ·0000094	+ ·0000068	+ ·0000026	− ·0000025	− ·0000078	− ·0000123	− ·0000153	− ·0000159	− ·0000148
11	+ ·0000028	+ ·0000053	+ ·0000074	+ ·0000088	+ ·0000091	+ ·0000082	+ ·0000062	+ ·0000033	− ·0000001	− ·0000034	− ·0000063	− ·0000083	− ·0000089	− ·0000089

21. The numbers of Table 26 are curved in dotted lines in fig. 48. The general correspondence of fact, as indicated by the thin-line curve, with our formula, as indicated by the dotted curves, cannot fail to arrest the attention of the reader; and, whatever be the physical conditions lying behind the phenomena, it cannot but be helpful towards their discovery that a compact mental grasp of the phenomena, such as the formula affords, should take the place of such notions of disconnected variations as are the first outcome of a study of the observations.

22. The numbers of Table 27 are curved in dotted lines in fig. 49, and on the same form are curved, in thin lines, the observed lunar diurnal variations of horizontal force, taken from Column 2 of Tables 5 to 8, for each of the eight phases of the moon. Here again the comparison of observation, as indicated by the thin-line curves, with the formula, through its representative dotted curves, shows remarkably close agreement.

23. That in their more prominent inflexions the dotted curves of fig. 48 are generally of smaller range than the thin-line curves is sufficiently explained by the fact that the typical variations for each phase are averages for three or four consecutive days, whilst the thin-line curves represent variations for first days, second days, third days, &c., after new moon—all single days of the moon's age. Indeed, it is easy to show that the typical variations $f_{c.1}(h)$, $f_{s.1}(h)$ should have been multiplied by

$$\frac{\frac{4\pi}{29\cdot53}}{\sin\frac{4\pi}{29\cdot53}} = 1\cdot031$$ when derived from the observed variations at the quarters, and by

$$\frac{\frac{3\cdot39\pi}{29\cdot53}}{\sin\frac{3\cdot39\pi}{29\cdot53}} = 1\cdot022$$ when derived from the observed variations at the eighths phases;

and similarly that $f_{c.2}(h), f_{s.2}(h)$ should have been multiplied by $\frac{\frac{8\pi}{29\cdot53}}{\sin\frac{8\pi}{29\cdot53}} = 1\cdot132$ and

by $\frac{\frac{6\cdot78\pi}{29\cdot53}}{\sin\frac{6\cdot78\pi}{29\cdot53}} = 1\cdot092$ in the same two cases respectively. The inadvertent omission to apply these factors is most influential in respect of the larger of them (1·132 and 1·092), since the range of the $f_{c.2}(h), f_{s.2}(h)$ variations is much greater than that of the $f_{c.1}(h)$ and $f_{s.1}(h)$.

24. In the case of the horizontal force curves (fig. 49) the factors would, if applied, affect both the thick-line and the thin-line curves alike, the latter representing the data from which the former were constructed; here, accordingly, the two sets of curves approach more nearly to identity of range.

25. The thick lines of figs. 48 and 49 have been drawn to represent the two last terms alone of our formula, viz., of

$$f_{c\cdot 2}(h)\cos 2\left(\frac{2\pi}{\mathrm{P}}t\right)+f_{s\cdot 2}\sin 2\left(\frac{2\pi}{\mathrm{P}}t\right).$$

Noticing that they deviate but little from the dotted lines, it may be inferred that the bulk of the systematic part of the variations is embraced by the restricted formula; and accordingly it is to that part of the variation that goes through a cycle of change in half a lunation that attention should first be directed in seeking for efficient physical causes.

Trevandrum.

26. Having thus found that order and system of a simple kind pervade the manifestations of minute lunar variations of magnetic force at Bombay, we turn with interest to records of a similar nature obtained at the near, but more nearly magnetically equatorial, station of Trevandrum. In a volume entitled 'Observations of Magnetic Declination made at Trevandrum and Agustia Malley in the Observatories of H. H. the Maharajah of Travancore,' and published by H. S. King and Co., of London, in 1874, the late John Allan Broun discussed the declination observations made at Trevandrum during the ten years 1854 to 1864, and—by a method of his own, which is, in essential points, similar to that described in the preceding pages—obtained values of the lunar diurnal variations for each month of the year and at each of the four quarters of the moon. Curves representing these variations appear as Plate 6 of the volume, and a glance at those for the months December and January, which—as at Bombay—have the largest range, is sufficient to show that our formula would correctly represent their principal features at each of the four quarters of the moon; indeed, those curves have a close resemblance to the corresponding winter curves for Bombay. An inspection of the curves for all the months leads to a suspicion that a period of ten years is scarcely sufficient to secure a counterbalancing, in all months, of casual irregularities; but, by combining together the variations of those consecutive months, the curves of which are similar in character, greater regularity is obtained, and the nature of the change of the typical variations with change of season is more distinctly brought out. The grouping of months adopted with these objects in view was as follows, viz.:—(1) December and January; (2) February and March; (3) April to June; (4) July to September; and (5) October and November. The average lunar diurnal variations for these several groups of months were calculated from the variations for individual months given in Tables XXXVII. to XL. (pages 125 and 126) of the book referred to. These average variations, which were expressed in minutes of arc, were then converted into force by multiplying by an adopted value of the horizontal force at Trevandrum (0·37141 C.G.S. unit), and by sine 1′, with results that are exhibited in the following Tables.

TABLE 28.—Declination. Trevandrum.

Twenty-fifths of Trevandrum lunar day.	0	1	2	3	4	5	6	7	8	9	10
New Moon	+·000054	+·000030	+·000009	−·000015	−·000024	−·000026	−·000022	−·000011	−·000006	−·000002	−·000001
First Quarter	+·000014	+·000011	+·000002	+·000005	·000000	−·000004	−·000007	−·000006	−·000007	−·000005	+·000002
Full Moon	+·000003	+·000005	+·000005	·000000	−·000007	−·000017	−·000025	−·000026	−·000021	−·000009	+·000002
Last Quarter.	+·000023	+·000026	+·000016	−·000002	−·000020	−·000027	−·000037	−·000032	−·000019	−·000001	+·000019

TABLE 29.—Declination. Trevandrum.

Twenty-fifths of Trevandrum lunar day.	0	1	2	3	4	5	6	7	8	9	10
New Moon	+·000015	+·000012	+·000001	−·000010	−·000019	−·000022	−·000016	−·000014	−·000009	−·000008	−·000002
First Quarter	+·000014	+·000005	−·000001	−·000002	−·000004	−·000005	−·000008	−·000007	−·000006	−·000004	·000000
Full Moon	+·000002	−·000002	−·000005	−·000010	−·000013	−·000015	−·000017	−·000015	−·000008	+·000003	+·000019
Last Quarter.	+·000007	+·000003	−·000009	−·000017	−·000023	−·000022	−·000011	−·000001	+·000006	+·000013	+·000018

TABLE 30.—Declination. Trevandrum.

Twenty-fifths of Trevandrum lunar day.	0	1	2	3	4	5	6	7	8	9	10
New Moon	−·000015	−·000018	−·000013	−·000006	·000000	+·000005	+·000005	+·000003	+·000001	+·000002	+·000003
First Quarter	−·000003	−·000003	·000000	·000000	−·000003	−·000006	−·000004	−·000006	−·000004	−·000004	−·000002
Full Moon	+·000008	+·000005	+·000004	·000000	·000000	−·000001	−·000001	+·000003	+·000006	+·000006	+·000002
Last Quarter.	−·000001	−·000005	−·000011	−·000009	−·000010	−·000006	+·000001	+·000006	+·000009	+·000010	+·000001

TABLE 31.—Declination. Trevandrum.

Twenty-fifths of Trevandrum lunar day.	0	1	2	3	4	5	6	7	8	9	10
New Moon	−·000022	−·000017	−·000004	+·000003	+·000007	+·000010	+·000006	+·000002	+·000001	−·000004	−·000002
First Quarter	+·000001	+·000004	+·000003	+·000002	+·000004	+·000004	+·000003	+·000001	+·000001	+·000001	−·000003
Full Moon	·000000	+·000004	+·000008	+·000008	+·000011	+·000013	+·000019	+·000018	+·000011	+·000001	−·000007
Last Quarter.	−·000006	−·000004	+·000006	+·000013	+·000008	+·000008	+·000009	+·000005	·000000	−·000007	−·000002

December and January.

11	12	13	14	15	16	17	18	19	20	21	22	23	24*
·000000	+·000003	+·000008	+·000006	+·000003	−·000005	−·000015	−·000029	−·000033	−·000020	−·000004	+·000014	+·000037	+·000051
+·000009	+·000022	+·000032	+·000032	+·000013	−·000012	−·000034	−·000040	−·000037	−·000027	−·000006	+·000008	+·000019	+·000015
+·000024	+·000040	+·000033	+·000025	+·000008	−·000006	−·000010	−·000010	−·000009	−·000005	−·000002	−·000002	·000000	+·000004
+·000022	+·000015	+·000009	+·000003	+·000001	·000000	−·000003	−·000008	−·000008	−·000007	−·000004	+·000002	+·000010	+·000024

February and March.

11	12	13	14	15	16	17	18	19	20	21	22	23	24
+·000001	+·000004	+·000008	+·000010	+·000012	+·000008	+·000002	−·000007	−·000007	−·000006	+·000001	+·000012	+·000016	+·000021
+·000008	+·000011	+·000012	+·000006	+·000002	−·000013	−·000023	−·000020	−·000016	−·000005	+·000008	+·000017	+·000015	+·000017
+·000026	+·000019	+·000012	+·000005	+·000001	−·000005	−·000005	−·000002	+·000001	+·000002	−·000001	+·000003	+·000003	+·000004
+·000016	+·000010	+·000005	+·000002	+·000003	+·000004	·000000	−·000003	−·000006	−·000004	−·000002	·000000	+·000004	+·000005

April to June.

11	12	13	14	15	16	17	18	19	20	21	22	23	24
+·000005	+·000004	+·000003	+·000004	+·000003	−·000003	−·000003	−·000004	+·000001	+·000006	+·000010	+·000011	+·000003	−·000007
−·000002	−·000006	−·000007	−·000008	−·000005	−·000002	+·000004	+·000008	+·000012	+·000013	+·000014	+·000008	+·000006	·000000
−·000004	−·000007	−·000006	−·000008	−·000007	−·000006	·000000	+·000001	−·000002	−·000001	−·000001	+·000001	+·000004	+·000005
−·000001	−·000002	−·000001	·000000	−·000003	−·000005	−·000003	−·000003	−·000001	+·000002	+·000006	+·000011	+·000009	+·000004

July to September.

11	12	13	14	15	16	17	18	19	20	21	22	23	24
−·000001	−·000001	+·000001	+·000001	+·000003	+·000006	+·000011	+·000015	+·000013	+·000009	+·000003	−·000007	−·000012	−·000018
−·000009	−·000013	−·000010	−·000007	+·000001	+·000006	+·000009	+·000009	+·000006	+·000004	·000000	−·000008	−·000008	−·000002
−·000010	−·000008	−·000014	−·000012	−·000007	+·000001	+·000003	+·000002	−·000010	−·000013	−·000009	−·000008	−·000002	−·000001
−·000006	−·000004	−·000003	+·000003	+·000003	+·000001	+·000004	−·000001	−·000003	−·000004	−·000003	−·000003	−·000006	−·000010

* In BROUN's mode of reduction the lunar day is divided into 25 parts.

TABLE 32.—Declination. Trevandrum.

Twenty-fifths of Trevandrum lunar day.	0	1	2	3	4	5	6	7	8	9	10
New Moon	+·000019	+·000014	+·000001	−·000007	−·000011	−·000015	−·000013	−·000010	−·000007	−·000004	−·000002
First Quarter	+·000006	·000000	−·000003	−·000005	−·000007	−·000008	−·000008	−·000009	−·000008	−·000012	−·000015
Full Moon	+·000007	+·000007	+·000009	+·000005	+·000002	−·000002	−·000008	−·000012	−·000014	−·000014	−·000008
Last Quarter	+·000003	+·000001	+·000003	−·000006	−·000008	−·000009	−·000008	−·000008	−·000006	+·000002	+·000007

27. By treating each of these sets of numbers in the manner described in paragraphs 8

TABLE

Twenty-fifths of a solar day after noon.		0	1	2	3	4	5	6	7	8	9	10
December and January	$f_{c\cdot1}(h)$	+·000009	·000000	−·000004	−·000008	−·000008	−·000008	−·000006	−·000002	−·000002	·000000	·000000
	$f_{s\cdot1}(h)$	−·000001	−·000001	+·000002	−·000001	·000000	−·000002	+·000001	+·000003	·000000	+·000002	+·000001
	$f_{c\cdot2}(h)$	+·000041	+·000028	+·000010	−·000008	−·000018	−·000017	−·000014	−·000009	−·000003	−·000002	·000000
February and March	$f_{c\cdot1}(h)$	·000000	+·000002	−·000001	−·000004	−·000007	−·000009	−·000008	−·000008	−·000005	−·000005	−·000003
	$f_{s\cdot1}(h)$	−·000005	−·000004	−·000001	+·000001	−·000001	+·000002	+·000003	+·000001	−·000002	−·000003	−·000003
	$f_{c\cdot2}(h)$	+·000013	+·000006	−·000004	−·000011	−·000014	−·000014	−·000010	−·000005	−·000002	−·000002	+·000001
April to June	$f_{c\cdot1}(h)$	−·000004	−·000006	−·000003	·000000	+·000001	+·000002	+·000003	+·000002	+·000001	+·000001	·000000
	$f_{s\cdot1}(h)$	+·000004	+·000003	+·000002	+·000001	+·000003	+·000001	−·000001	−·000001	+·000001	+·000002	+·000001
	$f_{c\cdot2}(h)$	−·000009	−·000012	−·000011	−·000007	−·000002	+·000002	+·000002	+·000001	·000000	+·000002	+·000003
July to September	$f_{c\cdot1}(h)$	−·000006	−·000002	+·000003	+·000003	+·000002	+·000003	+·000005	+·000007	+·000006	+·000002	+·000001
	$f_{s\cdot1}(h)$	·000000	+·000001	+·000002	−·000002	−·000002	+·000002	+·000002	+·000002	·000000	·000000	+·000001
	$f_{c\cdot2}(h)$	−·000012	−·000009	−·000002	+·000003	+·000005	+·000005	+·000001	−·000003	−·000004	−·000004	−·000003
October and November	$f_{c\cdot1}(h)$	+·000008	+·000004	−·000003	−·000004	−·000004	−·000006	−·000006	−·000005	−·000005	−·000004	−·000003
	$f_{s\cdot1}(h)$	·000000	+·000001	+·000002	+·000002	·000000	·000000	−·000002	−·000003	−·000004	−·000004	−·000004
	$f_{c\cdot2}(h)$	+·000010	+·000008	+·000002	−·000005	−·000008	−·000010	−·000007	−·000004	−·000002	·000000	+·000002

These variations are curved in figs. 17 to 21 and 38 to 47.

October and November.

11	12	13	14	15	16	17	18	19	20	21	22	23	24
+·000002	+·000004	+·000004	+·000006	+·000006	+·000003	−·000002	−·000008	−·000008	−·000005	·000000	+·000005	+·000010	+·000016
−·000009	−·000001	+·000013	+·000016	+·000018	+·000010	+·000006	−·000003	−·000008	−·000004	+·000001	+·000008	+·000009	+·000012
−·000004	+·000002	+·000006	+·000007	+·000005	−·000003	−·000003	−·000003	·000000	+·000002	+·000003	+·000005	+·000002	+·000008
+·000012	+·000011	+·000008	+·000006	+·000006	+·000002	+·000001	−·000001	−·000004	−·000004	−·000006	−·000003	·000000	+·000001

and 14, the several values of the typical variations $f(h)$ were obtained as follows :—

33.

11	12	13	14	15	16	17	18	19	20	21	22	23	24
−·000001	·000000	+·000002	·000000	·000000	−·000001	−·000002	−·000004	−·000004	+·000002	+·000006	+·000009	+·000012	+·000009
+·000001	+·000001	·000000	−·000001	−·000002	−·000002	−·000004	−·000001	+·000004	+·000006	+·000003	·000000	−·000005	−·000004
+·000003	+·000005	+·000006	+·000006	+·000003	−·000004	−·000013	−·000024	−·000029	−·000024	−·000010	+·000008	+·000027	+·000039
−·000001	+·000001	+·000004	+·000007	+·000010	+·000010	+·000008	+·000004	+·000004	+·000003	+·000002	·000000	−·000003	·000000
−·000002	−·000002	−·000001	−·000002	−·000002	−·000001	+·000002	+·000002	+·000003	+·000004	+·000007	+·000004	·000000	−·000002
+·000003	+·000005	+·000005	+·000004	+·000002	−·000001	−·000006	−·000010	−·000010	−·000005	+·000002	+·000014	+·000021	+·000020
·000000	−·000001	−·000002	·000000	·000000	−·000001	−·000001	−·000001	·000000	+·000001	+·000002	+·000004	+·000002	−·000001
−·000002	−·000001	−·000003	−·000004	−·000006	−·000006	−·000005	−·000004	−·000002	·000000	+·000002	+·000004	+·000006	+·000005
+·000005	+·000004	+·000004	+·000002	·000000	−·000003	−·000002	·000000	+·000003	+·000006	+·000008	+·000007	+·000001	−·000005
·000000	·000000	−·000001	−·000003	−·000002	−·000002	·000000	−·000001	−·000003	−·000003	−·000001	−·000002	−·000002	−·000005
+·000001	+·000002	+·000002	+·000002	+·000002	+·000001	·000000	−·000003	−·000003	−·000004	−·000004	−·000002	·000000	·000000
−·000002	−·000001	+·000001	+·000002	+·000003	+·000006	+·000010	+·000013	+·000012	+·000008	·000000	−·000007	−·000009	−·000011
−·000002	−·000002	−·000002	−·000001	·000000	·000000	−·000001	−·000001	+·000001	+·000004	+·000007	+·000008	+·000008	+·000008
−·000004	−·000003	−·000002	−·000002	−·000004	−·000007	−·000005	−·000001	+·000006	+·000007	+·000010	+·000008	+·000007	+·000003
+·000004	+·000006	+·000006	+·000007	+·000007	+·000006	+·000002	−·000004	−·000008	−·000009	−·000007	−·000002	+·000002	+·000006

28. On these curves we first remark that, as at Bombay, the variations of the $f_{c.1}(h)$ and $f_{s.1}(h)$ types are quite subordinate in magnitude to those of the $f_{c.2}(h)$ type; that the $f_{c.2}(h)$ curve is twice reversed in the course of the year, and that, whilst for December and January (fig. 17) it has a greater range than the corresponding winter curve (fig. 1) for Bombay, the reverse curve for July to September (fig. 20) has a smaller range than the corresponding autumn curve (fig. 4) for Bombay. The relation of the $f_{c.2}(h)$ curves of the two stations generally is such that the superposition of a constant variation of the character of the winter $f_{c.2}(h)$ curve, but of smaller range, upon the Bombay curves for each season would convert these into curves having the same principal characteristics as those of the same season for Trevandrum; and, in agreement with this, the reversal in the first half of the year occurs later at Trevandrum than at Bombay, and the reversal in the last half-year occurs earlier than at Bombay.

And here again we must insist on the reality of the variations now brought to light as true physical phenomena, which exhibit themselves not alone at a single point on the earth's surface, but are well marked in the observations of two stations that differ by nearly ten degrees in latitude, and the observatories at which were necessarily furnished with independent instruments, and had independent directors.

Postscript.

(Added September 23, 1886.)

The late John Allan Broun, in discussing the Trevandrum observations for the ten years 1854 to 1864, came to the conclusion that in the lunar diurnal variations of declination in each month of the year the amount of movement is, as in the solar diurnal variation, greater during the day than during the night; and the writer of this paper has shown in it that in each of four seasons of the year the same holds at Bombay with respect both to the declination and horizontal force. Whilst admitting, however, that Broun has priority of publication of this fact as regards the declination at Trevandrum, the writer avers that he has in no way been guided by Broun's previous investigation, nor was he aware of the existence of Broun's paper * when he made the discovery with respect to the declination at Bombay. Moreover, the writer's line of investigation was that of Sabine, modified by himself, and not that of Broun; and an account of the first results of its employment was read before the Royal Society on the 1st February, 1872, a date antecedent to the reading of Broun's paper before the Royal Society of Edinburgh, which did not take place till the 6th May, 1872. The writer also readily acknowledges that in the paper already referred to Broun enunciated some relations of the movements near sunrise in the lunar diurnal variations of declination in different months, which relations fit in well with the idea of a luni-solar variation when once that idea is conceived; but he finds

* 'Transactions of the Royal Society of Edinburgh,' vol. 26, page 735.

no trace of that idea in any of BROUN's writings. Neither does he find in BROUN's writings any of the following results that are described in the present paper, viz. :—

(1.) The discovery of relations subsisting between the lunar diurnal variation of declination *as a whole* at one phase of a lunation with the same at other phases in each season of the year [this as distinguished from the movements that occur *near sunrise*, and the nature of which was described by BROUN].

(2.) The discovery that a part, being the bulk of the whole, of the lunar diurnal variation runs through a cycle of change in a lunation ; to this part the name of the luni-solar variation is given.

(3.) A hypothesis is advanced—a hypothesis of phenomenal relations and not of physical causation—that the luni-solar variation of a given season is a combination of solar diurnal variations of constant types that go through cycles of wave-like change of amplitude in the periods of a lunation, half a lunation, &c., &c., and a formula is found to give expression to this hypothesis.

(4.) Following up this hypothesis, the observations of declination for the quarter November to January of the years 1846 to 1870 are divided into categories of solar days according to the age of the moon ; and the excess solar diurnal variation—the excess over the mean solar diurnal variation for the approximate full lunation—is entered in the Tables for each day, and the mean value is calculated for each category. Curves representing these mean excess solar diurnal variations for each day of the moon's age are found to agree fairly well with the formula expressing the luni-solar variation of the quarter November to January, a result which tends to confirm the hypothesis.

(5.) The mode of change, from season to season, of the character of the elements of the luni-solar variations is described.

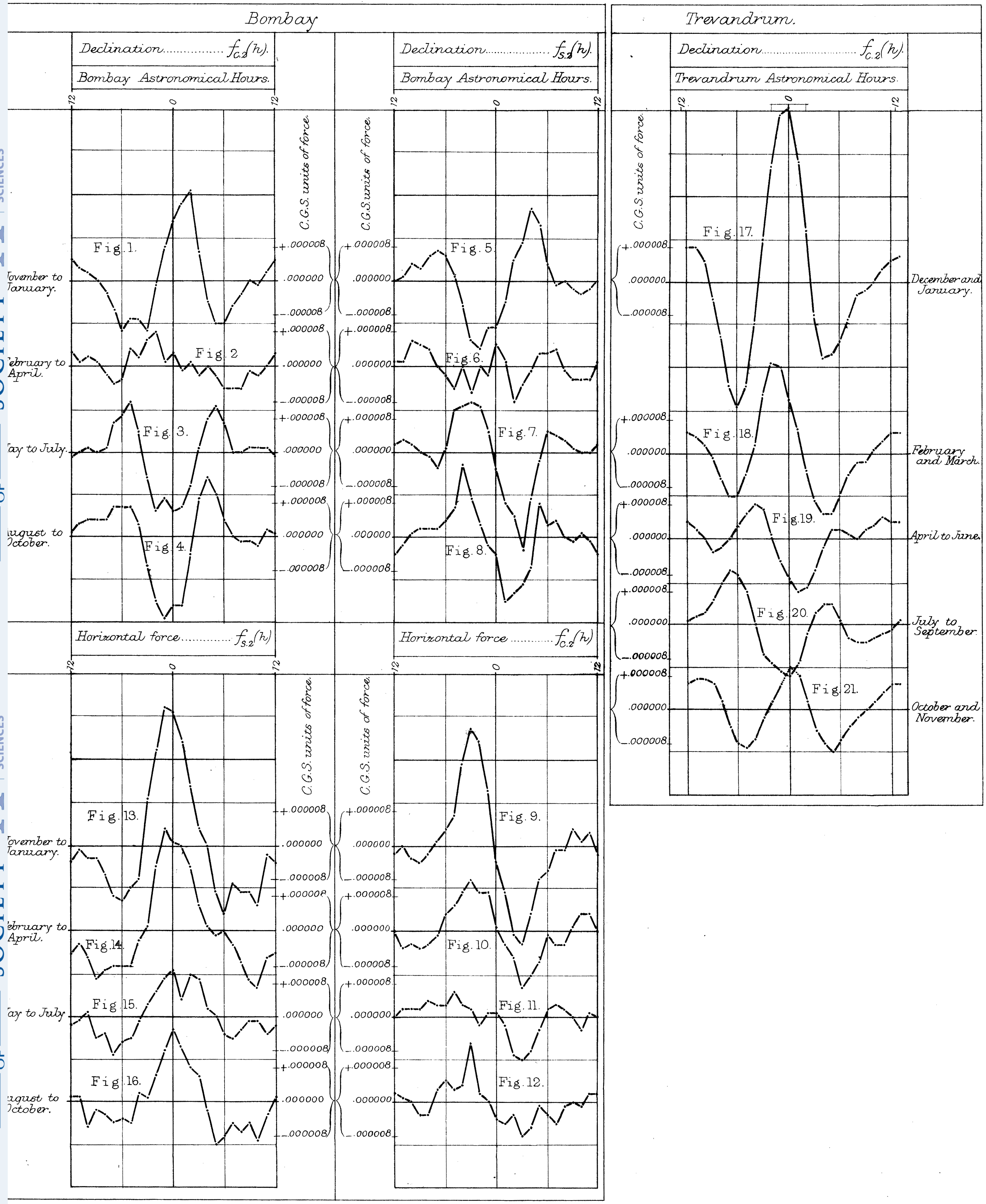
(6.) All the results, without exception, of the investigation with reference to the lunar variations of horizontal force must be placed on this list.

(7.) That the luni-solar variations of declination are related to those of the horizontal force in the following manner, viz. :—(*a*) In the winter season the luni-solar variation of declination due to any phase of the moon is similar to the luni-solar variation of horizontal force due to a phase later by one-eighth of a lunation ; and (*b*) in the summer and autumn seasons it is, on the other hand, the luni-solar variations of horizontal force that precede by an eighth of a lunation the similar variations of declination.

(8.) It is shown that, when the Bombay observations have pointed the way, BROUN's determinations of the lunar diurnal variations of declination at Trevandrum for the four quarters of the moon in each month of the year, when properly treated, support the hypothesis of result (3).

Colaba Observatory, Bombay.

Typical variations of declination and horizontal force at Bombay and of declination at Trevandrum.



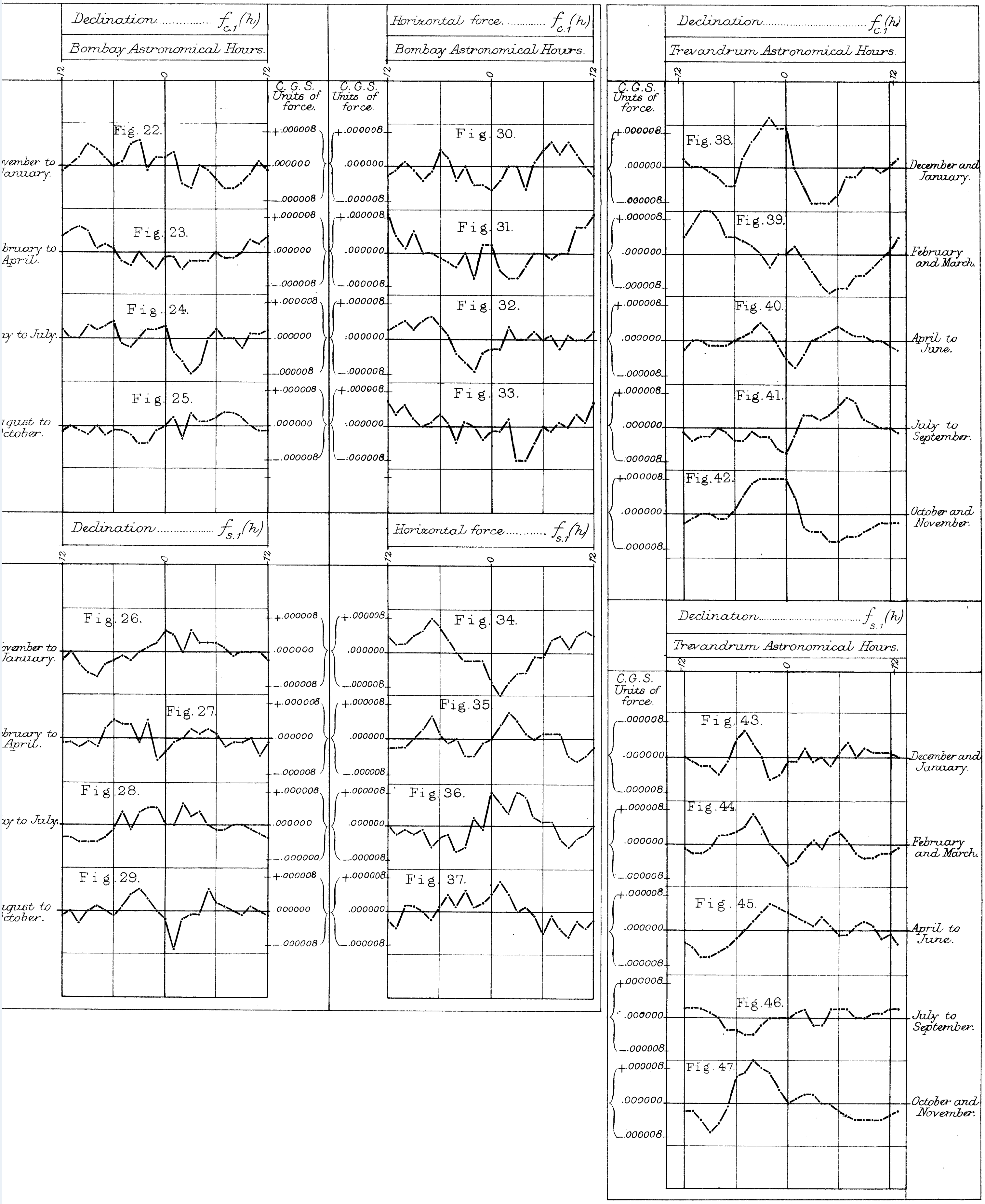
West, Newman & Co. lith.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Colaba Observatory, Bombay.

Typical variations of declination and horizontal force at Bombay.

Typical variations of declination at Trevandrum.



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Colaba Observatory, Bombay.

Curves representing, for a full lunation and for the quarter November to Janu
of the solar diurnal variation of declination which runs through a cycle of chan

Dotted curves .... calculated by the formula

$$f_{c.1}(h)\cos\frac{2\pi}{P}t + f_{s.1}(h)\sin\frac{2\pi}{P}t + f_{c.2}(h)\cos 2\frac{2\pi}{P}t + f_{s.2}(h)\sin 2\frac{2\pi}{P}t$$

h being the solar hour, t the age of the moon, and P the mean period

$f_{c.1}(h)$, $f_{s.1}(h)$, &c. each being a constant solar diurnal variation.

Thick-line curves ——·——·——·——·—— calculated by the last two terms only of the sam

Thin-line curves ——·——·—— variations given by observation.

An increase (upward movement) of one inch in the ordinates indicates an

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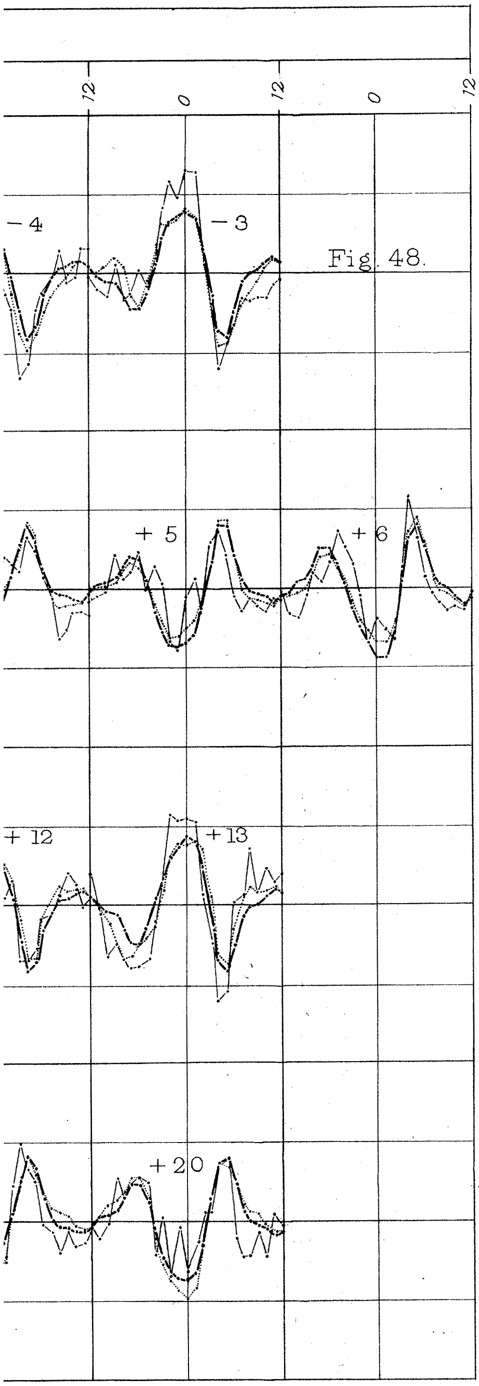

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Thin-line curves —·—·— variations given by observation.

An increase (upward movement) of one inch in the ordinates indicates an

easterly magnetic force of ·000048 C.G.S. unit.

The numbers with which the curves are marked represent the days before (–,

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

tes an increas of the

fore (−) or after (+) Ne Moon.

West, Newman & Co. lith.

Colaba Observatory, Bombay.

Curves representing, for a full lunation and for the quarter November to Januar the Solar diurnal variation of the horizontal force which runs through a cycle of change

Dotted curves .... *calculated by the formula*

$$f_{c.1}(h)\cos\frac{2\pi}{P}t+f_{s.1}(h)\sin\frac{2\pi}{P}t+f_{c.2}(h)\cos 2\frac{2\pi}{P}t+f_{s.2}(h)\sin 2\frac{2\pi}{P}t$$

h being the solar hour, t the age of the moon, and P the mean period of a

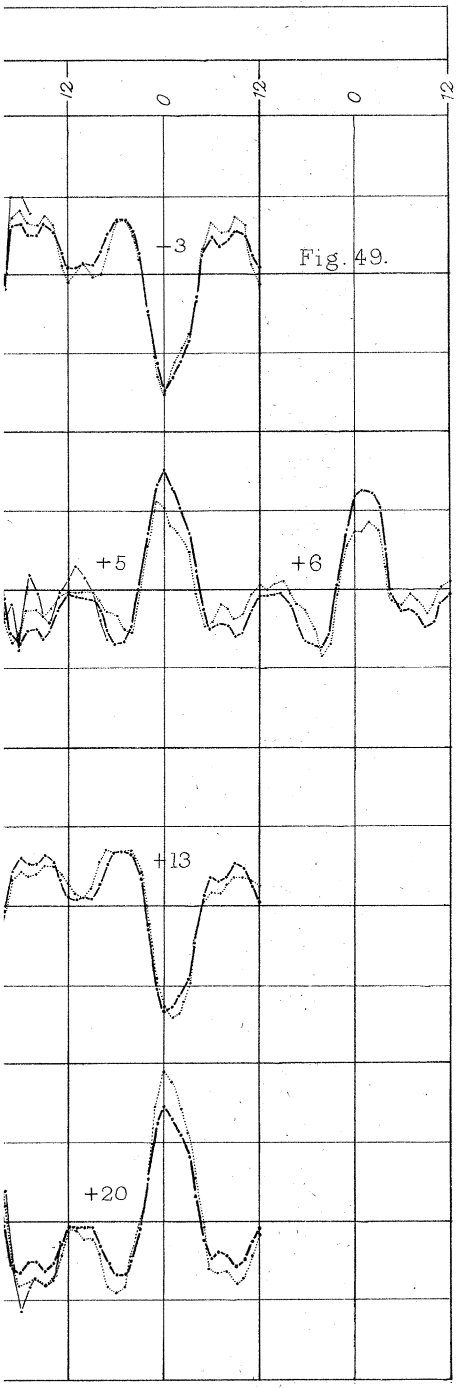
$f_{c.1}(h)$, $f_{s.1}(h)$, &c. each being a constant solar diurnal variation.

Thick line curves —·—·—·— *calculated by the last two terms only of the same*

Thin line curves —·—·— *variations given by observation.*

An increase (upward movement) of one inch in the ordinates indicates an increase

January, that part of
change in a lunation.



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PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Thin line curves —.—.— variations given by observation.

An increase (upward movement) of one inch in the ordinates indicates an increase

magnetic force of ·000048 C.G.S. unit.

The numbers with which the curves are marked represent the days before (−) or after

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

increase of the northerly

or after (+) New Moon.

West, [illegible] man & Co. lith.

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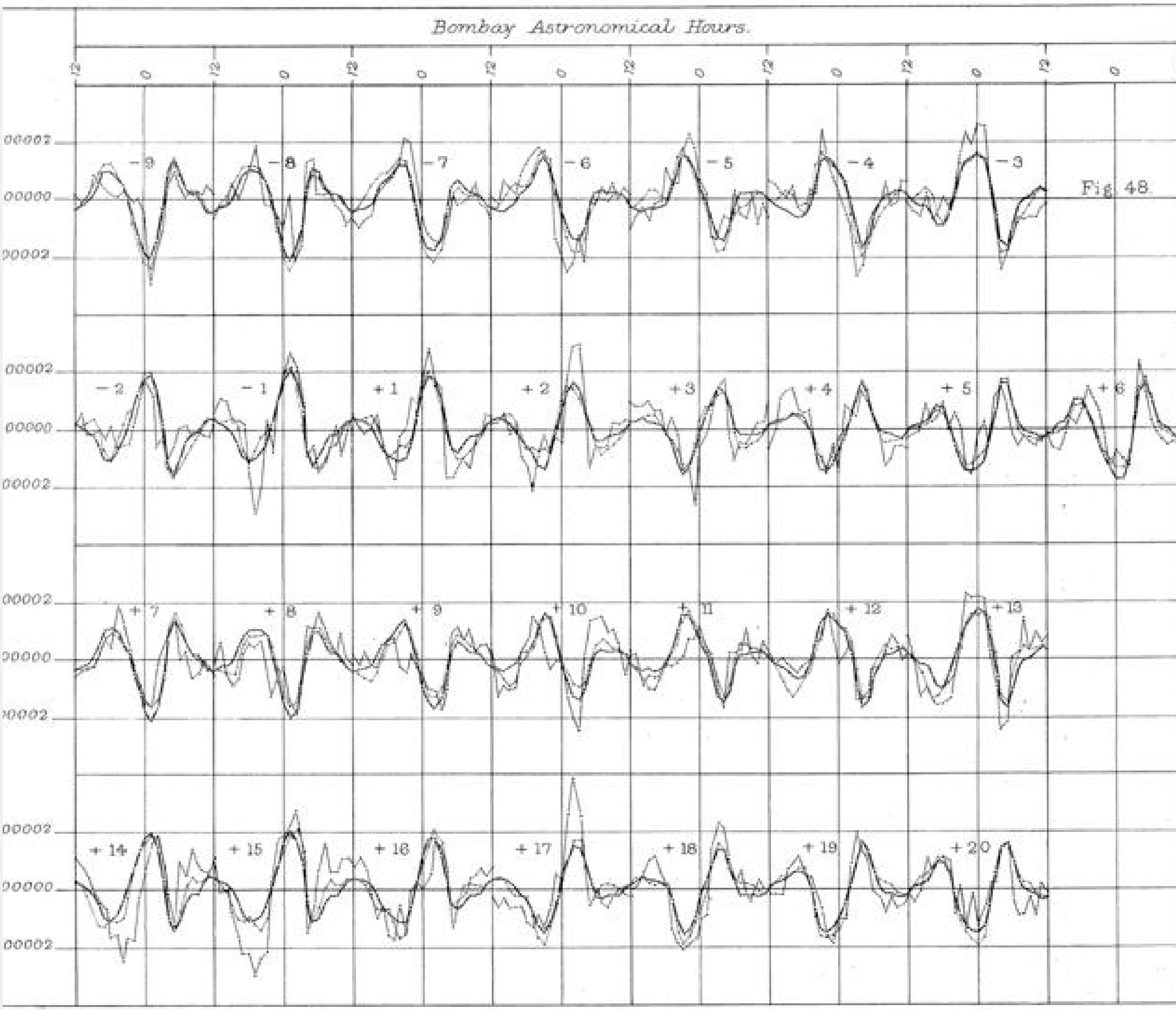
Colaba Observatory, Bombay.

Curves representing the lunar diurnal variations of declination for the four quarters of the Moon in each of four seasons of the year, and the typical variation for each season.

West, Newman & Co. lith.

Colaba Observatory, Bombay.

Curves representing, for a full lunation and for the quarter November to January, that part of the solar diurnal variation of declination which runs through a cycle of change in a lunation.



Dotted curves .... *calculated by the formula*

$$f_{c.1}(h)\cos\frac{2\pi}{P}t + f_{s.1}(h)\sin\frac{2\pi}{P}t + f_{c.2}(h)\cos 2\frac{2\pi}{P}t + f_{s.2}(h)\sin 2\frac{2\pi}{P}t$$

h being the solar hour, t the age of the moon, and P the mean period of a lunation; and $f_{c.1}(h)$, $f_{s.1}(h)$, &c. each being a constant solar diurnal variation.

Thick line curves ———————— *calculated by the last two terms only of the same formula.*

Thin line curves ———— *variations given by observation.*

An increase (upward movement) of one inch in the ordinates indicates an increase of the easterly magnetic force of ·000048 C.G.S. unit.

The numbers with which the curves are marked represent the days before (−) or after (+) New Moon.

West, Newman & Co. lith.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Colaba Observatory, Bombay.

Curves representing, for a full lunation and for the quarter November to January, that part of the Solar diurnal variation of the horizontal force which runs through a cycle of change in a lunation.

Dotted curves .... calculated by the formula

$$f_{c.1}(h)\cos\frac{2\pi}{P}t + f_{s.1}(h)\sin\frac{2\pi}{P}t + f_{c.2}(h)\cos 2\frac{2\pi}{P}t + f_{s.2}(h)\sin 2\frac{2\pi}{P}t$$

h being the solar hour, t the age of the moon, and P the mean period of a lunation; and $f_{c.1}(h)$, $f_{s.1}(h)$, &c. each being a constant solar diurnal variation.

Thick line curves ———————— calculated by the last two terms only of the same formula.

Thin line curves ———— variations given by observation.

An increase (upward movement) of one inch in the ordinates indicates an increase of the northerly magnetic force of ·000048 C.G.S. unit.

The numbers with which the curves are marked represent the days before (−) or after (+) New Moon.

West, [illegible]man & Co lith.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES